[This question paper contains 2 printed pages.]

Sr. No. of Question Paper : 6697 **D** **Your Roll No.....**

Unique Paper Code : 237152

Name of the Course : **B.Sc. (H) Statistics**

Name of the Paper : STHT – 102 : Calculus – I

Semester : I

Time : 3 Hours Maximum Marks : 75

Instructions for Candidates

1. Write your Roll No. on the top immediately on receipt of this question paper.
2. Attempt **six** questions in all.
3. Selecting **four** from **Section A** and **two** from **Section B.**

SECTION A

1. (a) If $y = (x^2 - 1)^n$, show that $(x^2-1)y_{n+2} + 2xy_{n+1} - n(n+1)y_n = 0$.

   (b) If $u = r\sin\theta\cos\phi$, $v = r\sin\theta\sin\phi$ and $w = r\cos\theta$, prove that $\frac{\partial(u, v, w)}{\partial(r, \theta, \phi)} = r^2\sin\theta$. (6,6½)

2. (a) Find the points of inflexion of the curve $y = (x-1)^3(x-5)$.

   (b) Find the asymptotes of the curve

   $x^3 + 2x^2y - xy^2 - 2y^3 + xy - y^2 - 1 = 0$ (6,6½)

3. (a) Locate the double points of the curve $x^3 + 2x^2 + 2xy - y^2 + 5x - 2y = 0$, and discuss their nature.

   (b) Trace the curve $r\cos\theta = a\cos 2\theta$. (6,6½)

P.T.O.

4. (a) Trace the curve $y^2 (a^2 - x^2) = x^4$.

(b) If $u = f(r)$, where $r = \sqrt{x^2 + y^2 + z^2}$, show that

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = f''(x) + \frac{1}{r} f'(x).$$ (6,6½)

5. (a) Obtain maximum and minimum value of the function $u = xy + \frac{a^3}{x} + \frac{a^3}{y}$.

(b) It $u = \tan^{-1} \frac{y}{x}$, show that $x^2 \frac{\partial^2 u}{\partial x^2} + 2xy \frac{\partial^2 u}{\partial x \partial y} + y^2 \frac{\partial^2 u}{\partial y^2} = 0$. (6,6½)

SECTION B

6. Solve the following differential equations :

(a) $p - \frac{1}{p} = \frac{x}{y} - \frac{y}{x}$, where $p = \frac{dy}{dx}$.

(b) $\left(\frac{e^{-2\sqrt{x}}}{\sqrt{x}} - \frac{y}{\sqrt{x}} \right) \frac{dx}{dy} = 1$

(c) $\left(1 + e^{x/y}\right) dx + e^{x/y} \left(1 - \frac{x}{y}\right) dy = 0$ (4,4,4½)

7. (a) Show that the equation $\frac{2x}{y^3} dx + \frac{y^2 - 3x^2}{y^4} dy = 0$ is exact and hence find its solution.

(b) Solve the following differential equations :

(i) $p = \tan(y - xp)$,

(ii) $y = xp^2 + p$,

where $p = \frac{dy}{dx}$. (4½,8)

8. Solve the following differential equations :

(i) $(D^2 + 5D + 6)y = e^{-2x} + \sin x$

(ii) $(D^2 + 2D + 1)y = x \cos^2 x$

(iii) $(D^3 + 2D^2 + D)y = e^x \sin 2x$ (4,4,4½)