[This question paper contains 4 printed pages.]

Sr. No. of Question Paper : 6698 **D** **Your Roll No...............**

Unique Paper Code : 237153

Name of the Course : **B.Sc. (H) Statistics**

Name of the Paper : STHT–103 : Algebra – I

Semester : I

Time : 3 Hours Maximum Marks : 75

Instructions for Candidates

1. Write your Roll No. on the top immediately on receipt of this question paper.
2. Attempt **six** questions in all, selecting **three** questions from each Section.

SECTION I

1. (a) If α, β, γ are the roots of the equation $x^3 + 7x^2 + 3x - 5 = 0$, then find the value of

(i) $\sum \frac{\beta^2 + \gamma^2}{\beta\gamma}$

(ii) $(\beta + \gamma)(\gamma + \alpha)(\alpha + \beta)$

(iii) $\sum \frac{1}{\alpha^2}$

(b) Solve the equation $3x^4 - 40x^3 + 130x^2 - 120x + 27 = 0$, given that the product of two of its roots is equal to the product of the other two.

(c) Form an equation whose roots are greater by 4 than the roots of the equation $x^4 - 6x^3 - 38x^2 - 3x + 17 = 0$. (4½,5,3)

2. (a) By substituting

$$z = \cos 2\theta + i\sin 2\theta,$$

$$a = \cos 2\alpha + i\sin 2\alpha,$$

$$b = \cos 2\beta + i\sin 2\beta,$$

in the identity

$$\frac{1}{(z-a)(z-b)} = \frac{1}{(a-b)}\left(\frac{1}{(z-a)} - \frac{1}{(z-b)}\right),$$

show that

$\sin(\alpha-\beta)\cos(\alpha+\beta+2\theta) + \sin(\beta-\theta)\cos(\beta+\theta+2\alpha) + \sin(\theta-\alpha)\cos(\alpha+\theta+2\beta) = 0.$

(b) Prove that the equation

$$\cos 2\phi + p\cos\phi + q\sin\phi + r = 0,$$

has in general, four solutions $\phi_1, \phi_2, \phi_3, \phi_4$ between 0 and 2π. Also show that $\phi_1 + \phi_2 + \phi_3 + \phi_4$ is a multiple of 2π.

(c) Sum the series :

$\cos\theta + x\cos 2\theta + x^2\cos 3\theta + \ldots$ to n terms. (5,4½,3)

3. (a) If n is a positive integer, then prove that

(i) $n! < \left(\frac{n+1}{2}\right)^n$

(ii) $n^n\left(\frac{n+1}{2}\right)^{2n} > (n!)^3$

(b) State and prove Cauchy Schwartz inequality.

(c) If a, b, c > 0 and $a \neq b \neq c$, then show that :

$(a+b)^3 + (b+c)^3 + (c+a)^3 \geq \frac{8}{9}(a+b+c)^3.$ (4,4½,4)

4. (a) Find the modulus and the argument of the complex number :

$$\frac{(\sin\alpha + i\cos\alpha)^4}{(\cos\alpha - i\sin\alpha)^3}$$

(b) Prove that

$128 \sin^3\theta \cos^5\theta = -\sin 8\theta - 2\sin 6\theta + 2\sin 4\theta + 6\sin 2\theta$.

(c) If α, β, γ be the roots of the equation $x^3 + px + q = 0$ then form an equation whose roots are $(\beta-\gamma)^2$, $(\gamma-\alpha)^2$, $(\alpha-\beta)^2$. (3½,4½,4½)

SECTION II

5. (a) Prove that the only matrices which are commutative for multiplication, with a diagonal matrix with distinct diagonal elements, are diagonal matrices.

(b) If A is a real skew-symmetric matrix such that

$$A^2 + I = O,$$

show that A is orthogonal and is of even order.

(c) Find the value of $adj(P^{-1})$ in terms of P where P is a non-singular matrix and hence show that $adj(Q^{-1}BP^{-1}) = PAQ$ given that $adj(B) = A$ and $|P| = |Q| = 1$. (3,4½,5)

6. (a) If A is Hermitian (Skew-Hermitian) then show that $B^{\theta}AB$ is Hermitian (Skew-Hermitian).

(b) If $F(\alpha) = \begin{pmatrix} \cos\alpha & -\sin\alpha & 0 \\ \sin\alpha & \cos\alpha & 0 \\ 0 & 0 & 1 \end{pmatrix}$, $G(\beta) = \begin{pmatrix} \cos\beta & 0 & \sin\beta \\ 0 & 1 & 0 \\ -\sin\beta & 0 & \cos\beta \end{pmatrix}$, then show that the inverse of the matrix $F(\alpha)\,G(\beta)$ is $G(-\beta)\,F(-\alpha)$.

(c) Use the inverse of the matrix to solve the given system of equations

$$2x - y + 3z = 9$$
$$x + y + z = 6$$
$$x - y + z = 2$$

(3,5,4'/2)

7. (a) If x, y, z are different and

$$A = \begin{vmatrix} x & x^2 & 1+x^3 \\ y & y^2 & 1+y^3 \\ z & z^2 & 1+z^3 \end{vmatrix} = 0,$$

then show that $1 + xyz = 0$.

(b) Prove that the determinant of a skew-symmetric matrix of even order is the square of the polynomial function of its elements.

(c) If A and B are two matrices of the same order $p \times q$, then show that

$$(A + B)^\theta = A^\theta + B^\theta.$$ (4½,5,3)

8. (a) Express the determinant

$$\begin{vmatrix} 1 & \cos(\beta-\alpha) & \cos(\gamma-\alpha) \\ \cos(\alpha-\beta) & 1 & \cos(\beta-\gamma) \\ \cos(\alpha-\gamma) & \cos(\beta-\gamma) & 1 \end{vmatrix}$$

as a product of two determinants. Hence evaluate it.

(b) If A is an idempotent matrix and A + B = I, then show that B is an idempotent matrix and AB = BA = O.

(c) If A and B are two $n \times n$ non-singular matrices, then show that

adj(AB) = adj(B) . adj(A) (6,3,3½)