[This question paper contains 4 printed pages.]

Sr. No. of Question Paper : 6699 **D** **Your Roll No.....**

Unique Paper Code : 237101

Name of the Course : **B.Sc. (Hons.) Statistics – First Year**

Name of the Paper : STHT – 104 : Probability and Statistical Methods – I

Semester : I

Time : 3 Hours Maximum Marks : 75

Instructions for Candidates

1. Write your Roll No. on the top immediately on receipt of this question paper.
2. Attempt **6** questions in all.
3. Q. No. **1** is compulsory.
4. Attempt **5** more questions selecting atleast **2** questions from each section.

1. (a) Fill in the blanks :

   (i) Mean Deviation is least when taken about _____ .

   (ii) If $\beta_2 < 3$, the distribution is said to be _____ .

   (iii) 10 is the mean of 7 observations and 5 is the mean of 3 observations. The mean of the combined set is _____ .

   (iv) If attributes A and B are completely associated then _____ .

   (v) If A and B are mutually exclusive events, then $P(A \cap B) =$ _____ .

   (b) Variance of first n natural numbers is _____ .

   (c) Suppose P(A)=0.7, P(B)=0.5 and $P(\bar{A} \cap \bar{B}) = 0.1$. Find

   (i) $P(A \cap B)$

   (ii) $P(A \mid B)$

P.T.O.

(d) If A and B are two mutually exclusive events, then show that

$$P(A|\bar{B}) = \frac{P(A)}{[1 - P(B)]}.$$

(e) Examine whether A and B are independent, positively associated or negatively associated given that :

$(A) = 490$, $(AB) = 294$, $(\alpha) = 57$ and $(\alpha\beta) = 380$.

(f) If 25% of the items are less than 10 and 25% are more than 40, then the coefficient of quartile deviation is _____. (5,2,2,2,2,2)

SECTION – A

2. (a) The mean and the standard deviation of a given variable X are m and σ respectively. If the deviations are small compared with the value of the mean, show that

$$\text{Mean}\left(\frac{1}{\sqrt{x}}\right) = \frac{1}{\sqrt{m}}\left(1 + \frac{3\sigma^2}{8m^2}\right) \text{ approximately.}$$

(b) Find the mean deviation about mean and standard deviation of A.P. a, a+d, a+2d, ..., a+2nd and verify that standard deviation is greater than mean deviation about mean. (6,6)

3. (a) Discuss the principle of least squares. Derive the normal equations for fitting the curve $Y = ab^x$ to the given set of n points $\{(x_i, y_i), i = 1,2 .... n\}$.

(b) Define Pearsonian coefficients β_1, β_2, γ_1, γ_2 and discuss their utility in statistics. Also prove that for any discrete distribution $\beta_2 > 1$. (6,6)

4. (a) Let r be the range and s be the standard deviation of a set of observations $x_1, x_2, ... x_n$. Prove that $s \leq r$.

(b) Given the following ultimate class frequencies, find the frequencies of positive class :

$(ABC) = 149$, $(AB\gamma) = 738$, $(A\beta C) = 225$, $(A\beta\gamma) = 1196$, $(\alpha BC) = 204$, $(\alpha B\gamma) = 1762$, $(\alpha\beta C) = 171$, $(\alpha\beta\gamma) = 21842$. (6,6)

5. (a) In an anti-malarial campaign in a certain area, quinine was administered to 812 persons out of a total population of 3248. The number of fever cases is shown below :

Treatment	Fever	No Fever
Quinine	20	792
No Quinine	220	2216

Discuss the usefulness of quinine in checking malaria.

(b) In a frequency table, the upper boundary of each class interval has a constant ratio to the lower boundary. Show that the geometric mean G may be expressed by the following formula :

$$\log G = x_0 + \frac{c}{N}\sum_i f_i(i-1),$$

where x_0 is the logarithm of the mid value of the first interval and c is the logarithm of the ratio between upper and lower boundaries. (6,6)

SECTION B

6. (a) Give the classical and statistical definitions of probability. What are the objections raised in these definitions ?

(b) If $p_1 = P(A)$, $p_2 = P(B)$, $p_3 = P(A \cap B)$ $(p_1, p_2, p_3 \geq 0)$, express the following in terms of p_1, p_2, p_3 :

(i) $P(\overline{A \cup B})$ (ii) $P(\overline{A} \cup \overline{B})$

(iii) $P(\overline{A} \cap B)$ (iv) $P(\overline{A} \cup B)$

(v) $P(\overline{A} \cap \overline{B})$ (vi) $P(A \cap \overline{B})$ (6,6)

P.T.O.

7. (a) For any three events A, B and C, defined on the sample space S, prove that

$P(A \cup B \mid C) = P(A \mid C) + P(B \mid C) - P(A \cap B \mid C)$.

(b) In a random arrangement of the letters of the word 'COMMERCE', find the probability that all the vowels come together. (6,6)

8. (a) An urn contains four tickets marked with numbers 112, 121, 211, 222 and one ticket is drawn at random. Let $A_i (i = 1,2,3)$ be the event that i^{th} digit of the number of the ticket drawn is 1. Discuss the independence of the events A_1, A_2 and A_3.

(b) State Bayes' theorem.

The contents of urns I, II and III are as follows :

1 white, 2 black and 3 red balls,

2 white, 1 black and 1 red balls and

4 white, 5 black and 3 red balls.

One urn is chosen at random and two balls are drawn from it. They happen to be white and red. What is the probability that they come from urn III ? (6,6)

(200)