[This question paper contains 4 printed pages.]

Sr. No. of Question Paper : 1144 **G** **Your Roll No...............**

Unique Paper Code : 237101

Name of the Paper : Probability and Statistical Methods – I (STHT-104)

Name of the Course : **B.Sc. (Hons.) Statistics**

Semester : I

Duration : 3 Hours Maximum Marks : 75

Instructions for Candidates

1. Write your Roll No. on the top immediately on receipt of this question paper.
2. Attempt **6** questions in all.
3. Q. No. **1** is compulsory.
4. Attempt **5** more questions selecting atleast **2** questions from each Section.
5. Use of simple calculators is allowed.

1. (a) Fill in the blanks :

(i) Median = Mode + ? (Mean – Mode)

(ii) The total number of class frequencies of all orders, for n attributes is ________ .

(iii) A motor when travelling from rest travels the first twentieth of a mile at 6 miles per hour and the next three twentieths at respectively 8, 12, 24 miles per hour. His average speed is ________ .

(iv) The relation between β_1 and β_2 is given by ________ .

(v) Letters are drawn one at a time from a box containing the letters A, H, M, O, S, T, R, E and E. The probability that the letters in the order spell the word THE MOARSE is ________ .

P.T.O.

(b) In a frequency distribution, the coefficient of skewness based upon the quartiles is 0.6. If the sum of the upper and lower quartiles is 100 and median is 38 then find the value of the upper and lower quartiles.

(c) Suppose $P(A) = 0.7$, $P(B) = 0.5$ and $P(\bar{A} \cap \bar{B}) = 0.1$. Find $P(A \cap B)$ and test whether A and B are independent events.

(d) If $(1 + 3p)/3$, $(1 - p)/4$ and $(1 - 2p)/2$ are the probabilities of three mutually exclusive events, then find the set of all values of p.

(e) Define quartiles and deciles.

(f) For a distribution mean is 10 and variance is 16, $\gamma_1 = 1$ and $\beta_2 = 4$. Find μ_3 and μ_4. (5,2,2,2,2,2)

SECTION – A

2. (a) In a discrete series, if the deviations $X_i = x_i - M$ are small compared with the value of the mean M so that $\left(\frac{X_i}{M}\right)^3$ and higher powers of $\left(\frac{X_i}{M}\right)$ are neglected, then show that

(i) $G = M\left(1 - \frac{1}{2}\frac{\sigma^2}{M^2}\right)$,

(ii) $M^2 = G^2 - \sigma^2$,

where G is the geometric mean of the values $x_1, x_2, \ldots, x_n$ and σ^2 is the variance.

(b) Find the mean deviation about mean and standard deviation (S.D.) of A.P. a, a+d, a+2d,..., a+2nd and verify that S.D. is greater than mean deviation about mean. (6,6)

3. (a) Discuss the principle of least squares. Derive the normal equations for fitting the curve $Y=a*\exp(bX + cX^2)$ to the given set of n points $\{(x_i, y_i),\ i = 1,2 \ldots.\ n\}$.

(b) Given that (A) = (α) = (B) = (β) = (C) = (γ) = N/2 and that (ABC) = (αβγ), then show that 2(ABC) = (AB) + (AC) + (BC) − N/2. (6,6)

4. (a) Let r be the range and s be the standard deviation of a set of observations $x_1, x_2, \ldots x_n$. Prove that $s \le r$. Also prove that $S \le r\left(\frac{n}{n-1}\right)^{\frac{1}{2}}$, where

$$S = \left[\frac{1}{n-1}\sum_{i=1}^{n}(x_i - \bar{x})^2\right]^{\frac{1}{2}}$$

(b) Define Yule's coefficient of association (Q) and coefficient of colligation (Y). Prove that $Q = 2Y/(1 + Y^2)$. What is the range of values for Q ? (6,6)

5. (a) Establish the relationship between the moments about mean, in terms of moments about any arbitrary point A. What are the Sheppard's corrections to central moments.

(b) The first four moments about x = 4 are 1, 4, 10, and 45. Determine the corresponding moments (i) about the mean, and (ii) about zero. (6,6)

SECTION B

6. (a) Give the classical and statistical definitions of probability. What are the objections raised in these definitions ? Explain with suitable examples.

(b) Three newspapers A, B, and C are published in a certain city. It is estimated from the survey that of the adult population 20% read A, 16% read B, 14% read C, 8% read both A and B, 5% read both A and C, 4% read both B and C, 2% read all three. Find the percentage that read at least one of the papers ? Also find the percentage that read both A and B but does not read C ? (6,6)

7. (a) Let $A_1, A_2, \ldots, A_n$ be the events in the domain of probability function P, such that $P\left[\bigcup_{i=1}^{n} A_i\right] \leq \sum_{i=1}^{n} P[A_i]$. Using this relationship, prove that :

(i) $P\left[\bigcap_{i=1}^{n} A_i\right] \geq 1 - \sum_{i=1}^{n} P[\bar{A}_i]$, and

(ii) $P\left[\bigcap_{i=1}^{n} A_i\right] \geq \sum_{i=1}^{n} P[A_i] - (n-1)$.

(b) From a city population, the probability of selecting a male or a postgraduate is 7/10. The probability of selecting a male postgraduate is 2/5 and a male if postgraduate is already selected is 2/3. Find the probability of selecting (i) a non-postgraduate, (ii) a male and (iii) a postgraduate, if a male is first selected. (6,6)

8. (a) Define independent events, pairwise independence, and mutual independence.

A sample contains six points $E_1, E_2, E_3, E_4, E_5, E_6$ with the probabilities $P(E_1) = 0.20$, $P(E_2) = 0.05$, $P(E_3) = 0.30$, $P(E_4) = 0.10$, $P(E_5) = 0.10$ and $P(E_6) = 0.25$. Three events A, B and C are defined as follows :

$A = \{E_1, E_2, E_3\}$, $B = \{E_3, E_4\}$ and $C = \{E_5, E_6\}$

(i) Which pair of events A, B and C is/are mutually exclusive ?

(ii) Which pair of events A, B and C is/are independent ?

(iii) Are the events A, B and C mutually independent ?

(b) There are 2 bags A and B. A contains n white and 2 black balls and B contains 2 white and n black balls. One of the two bags is selected at random and two balls are drawn from it without replacement. If both the balls drawn are white and the probability that the bag A was used to draw the balls is 6/7, find the value of n. (6,6)