[This question paper contains 4 printed pages.]

Sr. No. of Question Paper : 1143 **G** **Your Roll No................**

Unique Paper Code : 237153

Name of the Paper : Algebra – I (STHT–103)

Name of the Course : **B.Sc. (Hons.) Statistics**

Semester : I

Duration : 3 Hours Maximum Marks : 75

Instructions for Candidates

1. Write your Roll No. on the top immediately on receipt of this question paper.
2. Attempt **six** questions in all, selecting **three** questions from each **Section.**

SECTION I

1. (a) If α, β, γ are roots of the equation $x^3 - 6x^2 + 11x - 6 = 0$, then find the value of

   (i) $\sum(\alpha^2 - 2)$

   (ii) $\sum(1-\alpha)(1-\beta)$

   (iii) $\sum(\alpha+\beta-\gamma)$

   (b) Find sum of 9^{th} power of roots of equation $x^3 + 3x - 1 = 0$.

   (c) Solve $x^3 - 9x^2 + 11x + 21 = 0$, given that the roots of the equation are in AP. (4½,4,4)

2. (a) If $x_r = \cos\left(\frac{\pi}{3^r}\right) + i\sin\left(\frac{\pi}{3^r}\right)$, r = 1, 2, 3, ... then find value of $\prod_{j=1}^{n} x_j$ hence evaluate $x_1x_2x_3..$

(b) Find all the values of $(1+i)^{\frac{1}{4}}$ and hence calculate their continued product.

(c) Using Demoivre's theorem, find all roots of equation
$x^5 + x^4 + x^3 + x^2 + x + 1 = 0$. (4½,4,4)

3. (a) Show that $1+\frac{1}{2}+\frac{1}{3}+...+\frac{1}{n} \le \sqrt{2n-1}$.

(b) If a, b, c and d are positive numbers, then show that

$a^5 + b^5 + c^5 + d^5 \ge abcd(a + b + c + d)$.

(c) If a, b, c represent the lengths of sides of a triangle, then prove that
$(a^2b+b^2c+c^2a)(ab^2+bc^2+ca^2) \ge 9a^2b^2c^2$. (4½,4,4)

4. (a) Solve $4x^4 - 24x^3 + 31x^2 + 6x - 8 = 0$, given that the sum of the two of its roots is zero.

(b) Show that $(1 + \cos\theta + i \sin\theta)^n + (1 + \cos\theta - i \sin\theta)^n = 2^{n+1} \cos^n\left(\frac{\theta}{2}\right)\cos\left(\frac{n\theta}{2}\right)$.

(c) Prove that $\sqrt{1}+\sqrt{2}+\sqrt{3}+...+\sqrt{n} \le n\sqrt{(n+1)/2}$. (4½,4,4)

SECTION – II

5. (a) Let A be a square matrix of order n with all elements equal to unity and B be a square matrix of order n with all diagonal elements equal to n and other elements equal to n-r, then show that

$A^2 = nA$. Deduce that $(B-rI)\,[B-(n^2 - nr + r)\,I] = 0$

(b) Show that A^2 is symmetric if either A is symmetric or A is skew symmetric.

(c) If A and B are two square matrices of order n, then show that

$$\text{adj}(AB) = \text{adj}B.\ \text{adj}A$$ (4½,4,4)

6. (a) If (l_1, m_1, n_1), (l_2, m_2, n_2) be the direction cosines of two perpendicular lines, then prove that the matrix product

$$\begin{pmatrix} l_1^2 & l_1m_1 & l_1n_1 \\ l_1m_1 & m_1^2 & m_1n_1 \\ l_1n_1 & m_1n_1 & n_1^2 \end{pmatrix}\begin{pmatrix} l_2^2 & l_2m_2 & l_2n_2 \\ l_2m_2 & m_2^2 & m_2n_2 \\ l_2n_2 & m_2n_2 & n_2^2 \end{pmatrix}$$

is a zero matrix.

(b) Show that every square matrix can be expressed uniquely as the sum of a Hermitian and a Skew Hermitian matrix.

(c) Use determinants to solve the following equations :

$$ax + by + cz = 1$$

$$a^2x + b^2y + c^2z = k$$

$$a^3x + b^3y + c^3z = k^2$$ (4½,4,4)

7. (a) Define Circulant Determinant. Show that if Δ is a circulant determinant, then

$$\Delta = \varphi(1)\ \varphi(\omega)\ \varphi(\omega^2) \ldots\ldots \varphi(\omega^{n-1}),$$

where $\varphi(x) = a_1 + a_2x + a_3x^2 + \ldots\ldots + a_nx^{n-1}$ and ω is an nth root of unity.

(b) If A is a symmetric matrix and B is a skew symmetric matrix, both of order n such that (A + B) is non singular and $C = (A + B)^{-1}(A - B)$, then prove that

(i) $C'(A + B)C = A + B$

(ii) $C'(A - B)C = A - B$

(iii) $C'AC = A$

(c) Define Unitary, Orthogonal and Idempotent matrices. Show that every non–singular idempotent matrix is an identity matrix. (4½,4,4)

8. (a) Prove that

$$\begin{vmatrix} a & b & c \\ b & c & a \\ c & a & b \end{vmatrix}^2 = \begin{vmatrix} 2bc-a^2 & c^2 & b^2 \\ c^2 & 2ca-b^2 & a^2 \\ b^2 & a^2 & 2ab-c^2 \end{vmatrix}$$

and is also equal to $(a^3 + b^3 + c^3 - 3abc)^2$.

(b) For each real number x, such that $-1 < x < 1$, let A(x) be a matrix defined as

$A(x) = (1 - x^2)^{-1/2}\begin{pmatrix} 1 & -x \\ -x & 1 \end{pmatrix}$, then show that

$A(x)\,A(y) = A(z)$, where $z = \dfrac{x+y}{1+xy}$. Deduce that $[A(x)]^{-1} = A(-x)$.

(c) Show that a skew symmetric determinant of order 4 is the square of the polynomial function of its elements. (4½,4,4)