[This question paper contains 2 printed pages.]

**Sr. No. of Question Paper : 1142** **G** **Your Roll No.....**

Unique Paper Code : 237152

Name of the Paper : Calculus – I (STHT–102)

Name of the Course : **B.Sc. (Hons.), STATISTICS**

Semester : I

Duration : 3 Hours Maximum Marks : 75

**Instructions for Candidates**

1. Write your Roll No. on the top immediately on receipt of this question paper.
2. Attempt **Five** questions in all.
3. Select **Three** questions from **Section-I** & **Two** questions from **Section-II.**

## SECTION – I

1. (a) Discuss the continuity of the function

$$f(x) = \begin{cases} 1, & x \le 0 \\ 3-x, & 0 < x \le 1 \\ \dfrac{4}{x+1}, & 1 < x \end{cases}$$

at $x = 0$ and $x = 1$.

(b) Verify that $\dfrac{\partial^2 z}{\partial x \partial y} = \dfrac{\partial^2 z}{\partial y \partial x}$ for $z = x^2 \tan^{-1}\dfrac{y}{x}$.

(c) If $p^2 = a^2 \cos^2\theta + b^2 \sin^2\theta$, prove that

$$p + \frac{d^2p}{d\theta^2} = \frac{a^2b^2}{p^3}$$ (5,5,5)

2. (a) Find the $n^{th}$ derivative of $y = \sin^{-1}\dfrac{2x}{x^2+1}$.

(b) Prove that if $y^3 - 3ax^2 + x^3 = 0$ then $\dfrac{d^2y}{dx^2} + \dfrac{2a^2x^2}{y^5} = 0$.

*P.T.O.*

(c) Find the maxima and minima of the function

$$f(t) = \sin t + \frac{1}{2}\sin 2t + \frac{1}{3}\sin 3t \quad \forall\, t \in [0, \pi]$$ (6,5,4)

3. (a) Find the asymptotes of the curve

$(y-a)^2(x^2-a^2) = x^4 + a^4$.

(b) Find the position and nature of the multiple points on the following curve

$(y^2 - a^2)^3 + x^4(2x + 3a)^2 = 0$.

(c) Find the points of inflexion of the curve

$x = a \tan t,\ y = a \sin t \cos t$. (5,5,5)

4. (a) Find the asymptotes of the curve

$r \cos 2\theta = a \sin 3\theta$.

(b) Trace the curve

$y^2(x + a) - x^2(3a - x) = 0$. (5,10)

## SECTION – II

5. Solve the following differential equations :

(a) $\frac{dy}{dx} + \frac{x-2y+5}{2x+y-1} = 0$

(b) $\left(x^3y^2 + xy\right)dx = dy$

(c) $x^3\frac{dy}{dx} - x^2y + y^4 \cos x = 0$ (5,5,5)

6. Solve the following differential equations :

(a) $p^3y^2 - 2xp + y = 0$

(b) $4p^3 + 3xp = y$

(c) $4y = x^2 + p^2$ (5, 5, 5)

7. Solve the following differential equations :

(a) $(D^2 - 2D + 1)y = (1 + e^{-x})^2$

(b) $(D^3 + D^2 + D + 1)y = x^5 - 2x^2 + x$

(c) $(D^4 - 1)y = x^2 \sin x$ (5,5,5)

(100)