[This question paper contains 4 printed pages.]

Sr. No. of Question Paper : 1797 **GC-3** **Your Roll No................**

Unique Paper Code : 32371109

Name of the Paper : Calculus

Name of the Course : **B.Sc. (Hons.) Statistics under CBCS**

Semester : I

Duration : 3 Hours Maximum Marks : 75

Instructions for Candidates

1. Write your Roll No. on the top immediately on receipt of this question paper.
2. Question No. **1** is compulsory.
3. From the remaining attempt **five** questions, selecting at least one from each section.

1. Attempt any **five** parts :

(a) Show that $\lim_{x\to 0}\frac{e^{\frac{1}{x}}-1}{e^{\frac{1}{x}}+1}$ does not exist.

(b) Evaluate $\lim_{x\to 0}\frac{x\cos x-\log(1+x)}{x^2}$.

(c) Prove that $\int_0^{\infty}\frac{x^c}{c^x}dx=\frac{\Gamma(c+1)}{(\log c)^{c+1}}$, $c>1$.

(d) Evaluate $\int_0^1\int_0^1\frac{1}{\sqrt{(1-x^2)(1-y^2)}}\,dx\,dy$.

P.T.O.

(e) Solve $(1 + y^2)dx = (\tan^{-1} y - x)dy$.

(f) $(D^2 - 3D + 2)y = 3\sin x$.

(g) Find partial differential equation of all planes a distance of a units from origin.

(h) Solve partial differential equation $(y - z)p + (z - x)q = x - y$. (5×3)

SECTION – I

2. (a) Determine the minimum value of $x^2 + y^2 + z^2$ subject to the condition

$$x + 2y - 4z = 5.$$

(b) If $y = \cos(m(\sin^{-1} x))$, show that $(1 - x^2)y_{n+2} - (2n + 1)xy_{n+1} + (m^2 - n^2)y_n = 0$ and hence evaluate $y_n(0)$. (6,6)

3. (a) If A, B and C are the angles of a triangle such that

$\sin^2 A + \sin^2 B + \sin^2 C = \sqrt{3}$, prove that $\frac{dA}{dB} = \frac{\tan B - \tan C}{\tan C - \tan A}$.

(b) Find the position and nature of the double points on the curve

$$(y - 2)^2 = x(x - 1)^2.$$ (6,6)

SECTION – II

4. (a) Prove that $\int_0^{\frac{\pi}{2}} \frac{d\theta}{\sqrt{a \cos^4\theta + b \sin^4\theta}} = \frac{\Gamma^2\left(\frac{1}{4}\right)}{4\sqrt{\pi}(ab)^{\frac{1}{4}}}$.

(b) Assuming the validity of differentiation under integral sign, prove that

$$\int_0^\infty e^{-x^2} \cos \alpha x \, dx = \frac{\sqrt{\pi}}{2} e^{-\frac{1}{4}\alpha^2} .$$ (6,6)

5. (a) Find the limit, when n trends to infinity, of the sum : $\sum_{r=1}^{n-1} \frac{1}{n}\sqrt{\frac{n+r}{n-r}}$.

(b) Change the order of integration in $\int_0^{3a} \int_{x^2/4a}^{3a-x} F(x,y)\,dy\,dx$ and hence evaluate when $F(x,y) = x + y$. (6,6)

SECTION – III

6. Solve the following differential equations :

(i) $(x^2y - 2xy^2)dx - (x^3 - 3x^2y)dy = 0$

(ii) $(1-x^2)\frac{dy}{dx} + 2xy = x(1-x^2)^{\frac{1}{2}}$ (6,6)

7. Solve any two of the following differential equations :

(i) $x^2\frac{d^2y}{dx^2} + 7x\frac{dy}{dx} + 13y = \log x$

(ii) $(1+2x)^2\frac{d^2y}{dx^2} - 6(1+2x)\frac{dy}{dx} + 16y = 8(1+2x)^2$

(iii) $(D^2 + 2D + 1)y = 2x + x^2$ (6,6)

SECTION – IV

8. Solve any two of the following partial differential equations :

(i) $(x^2 - y^2 - z^2)p + 2xyq = 2xz$

(ii) $x^2p^2 + y^2q^2 = z^2$

(iii) $(D^2 + DD' - 6D'^2)z = y \cos x$ (6,6)

9. (a) Solve $p^2 + q^2 = 1$ using variable separation method.

(b) Solve the partial differential equation $\frac{\partial^2 z}{\partial x^2} + x^2 \frac{\partial^2 z}{\partial y^2} = 0$. (6,6)