This question paper contains 7 printed pages]

Roll No. | | | | | | | | | | | |

S. No. of Question Paper : **1856**

Unique Paper Code : **237253** **C**

Name of the Paper : **ALGEBRA-II (STH-202)**

Name of the Course : **B.Sc. (Hons.) STATISTICS**

Semester : **II**

Duration : **3 Hours** Maximum Marks : **75**

(Write your Roll No. on the top immediately on receipt of this question paper.)

Attempt *six* questions in all. Question No. **1** is compulsory.

Attempt *five* more questions, selecting at least *two* questions from each Section. Use of simple calculator is allowed.

1\. (*a*) State whether the following statements are true *or* false :

(*i*) Inverse of $E_{ij}(k)$, $k \neq 0$ is $E_{ij}\left(\frac{1}{k}\right)$.

(*ii*) A is n-square matrix of rank $(n - 1)$, then its adjoint is of rank 1.

(*iii*) One of the characteristic roots of an orthogonal matrix is always zero.

(*iv*) A sum vector is a vector with all its components as unity.

(*v*) A division ring need not be an integral domain.

(*b*) Is the matrix $A = \begin{bmatrix} 0 & 1 & 3 \\ 1 & 2 & 3 \\ 3 & 1 & 1 \end{bmatrix}$ equivalent to I_3 ? Justify.

(*c*) Show that if $\lambda_1, \ldots\ldots, \lambda_n$ are n eigen values of a square matrix A of order 'n' then the eigen values of the matrix A^2 are $\lambda_1^2, \ldots\ldots \lambda_n^2$.

(*d*) Define :

(*i*) a real quadratic form;

(*ii*) discriminant of the quadratic form.

(*e*) Express (1, 2, 3) as a linear combination of (1, 1, 1), (2, –1, 1) and (1, –2, 5) in $V_3(R)$.

(*f*) Show that the set $q = \{a + \sqrt{2}\, b \ \forall a, b \in Q\}$ is a group with respect to addition. (5,2,2,2,2,2)

SECTION I

2. (*a*) Prove that every non-singular matrix can be reduced to the normal form by :

(*i*) E-row transformation only; and

(*ii*) E-column transformation only.

Hence or otherwise find the rank of the matrix :

$$A = \begin{pmatrix} 6 & 1 & 3 & 8 \\ 4 & 2 & 6 & -1 \\ 10 & 3 & 9 & 7 \\ 16 & 4 & 12 & 15 \end{pmatrix}.$$

(b) Discuss for all values of k, the solutions for the following system of equations :

$$2x + 3ky + (3k + 4)z = 0$$

$$x + (k + 4)y + (4k + 2)z = 0$$

$$x + 2(k + 1)y + (3k + 4)z = 0. \qquad (6,6)$$

3. (a) If A is a matrix of rank $(n - 1)$, prove that B, the adjoint of A has the characteristic equation $\lambda^n - \lambda^{n-1}\left(\Sigma_{i=1}^{n} b_{ij}\right) = 0$, where b_{ij} $(i, j = 1, 2 \ldots\ldots n)$ are the elements of B.

(b) Find the characteristic root of the matrix $A = \begin{pmatrix} 6 & -2 & 2 \\ -2 & 3 & -1 \\ 2 & -1 & 3 \end{pmatrix}$

and show that the characteristic vectors associated with its distinct characteristic roots are mutually orthogonal. (5,7)

4. (a) Reduce the following quadratic form $x_1x_2 + x_1x_3 + x_2x_3$ to the canonical form and hence find the rank, index and signature.

(*b*) Define elementary matrices. Obtain the inverses of elementary matrices. (6,6)

5. (*a*) Obtain the characteristic equation of the matrix :

$$A = \begin{pmatrix} 1 & 0 & 2 \\ 0 & 2 & 1 \\ 2 & 0 & 3 \end{pmatrix}.$$

Verify the Cayley Hamilton theorem. Hence or otherwise calculate its inverse.

(*b*) Prove that the definiteness of a quadratic form is invariant under non-singular linear transformation. (7,5)

SECTION II

6. (*a*) Discuss the algorithm for finding a generalized inverse of a given matrix. How will you find a symmetric generalized inverse for a symmetric matrix of order n ?

Let A be an $m \times n$ matrix of rank r and suppose A is partitioned as

$$A = \begin{pmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{pmatrix},$$

where A_{11} is $r \times r$ non-singular matrix.

Show that :

$$A \begin{bmatrix} \text{adj } A_{11} & 0 \\ 0 & 0 \end{bmatrix} A = |A_{11}| A.$$

(*b*) If $A = \begin{pmatrix} \alpha & \beta \\ \gamma & \delta \end{pmatrix}$, where α, β, γ and δ are sub-matrices and $|\alpha| \neq 0$, then find the inverse of A by method of partitioning. (6,6)

7. (*a*) Show that the set F of all matrices of the form $\begin{pmatrix} a & b \\ -b & a \end{pmatrix}$; where $a, b \in R$ is a field, with respect to matrix addition and matrix multiplication.

(*b*) Define linear combination of vectors and linear span of a set. Compute the angle between the vectors a = [4, 7, 9, 1, 3]; b = [2, 1, 1, 6, 8]. (6,6)

8. (*a*) Given two linearly independent vectors a_1, a_2 from E^2, obtain a formula for any vector x in E^2 as a linear combination of a_1, a_2. Hence express x = [9, 10] as a linear combination of a_1 = [2 1]; a_2 = [3 5].

(*b*) If the n-component vectors a, b, c are linearly independent, show that $a + b, b + c, a + c$ are also linearly independent. Is this true for $a - b, b + c, a + c$?

(*c*) Given the basis vectors e_1, [0, 1, 1], e_2 for E^3. Which vectors can be removed from the basis and be replaced by b = [4, 3, 3] while still maintaining a basis ? (6,3,3)