This question paper contains 4 printed pages]

Roll No. | | | | | | | | | | | |

S. No. of Question Paper : **1855**

Unique Paper Code : **237252** **C**

Name of the Paper : **Calculus II (STHT-201)**

Name of the Course : **B.Sc. (H) Statistics**

Semester : **II**

Duration : **3 Hours** Maximum Marks : **75**

(Write your Roll No. on the top immediately on receipt of this question paper.)

Attempt *two* questions from each Section.

Section I

1\. (*a*) If the equation :

$$ax^2 + 2hxy + by^2 + 2gx + 2fy + c = 0$$

represents two straight lines, prove that the product of the lengths of the perpendiculars from the origin on these lines is :

$$\frac{c}{\sqrt{(a-b)^2 + 4h^2}}.$$

(*b*) Show that the straight lines given by the equation :

$$(ax + by)^2 - 3(bx - ay)^2 = 0$$

form with the line $ax + by + c = 0$ an equilateral triangle, whose area is :

$$c^2 / [\sqrt{3}(a^2 + b^2)].$$ 6,6½

P.T.O.

2. (*a*) Tangents are drawn from the point (h, k) to the circle $x^2 + y^2 = a^2$. Prove that the area of the triangle formed by them and the straight line joining their points of contact is :

$$\frac{a(h^2 + k^2 - a^2)^{3/2}}{(h^2 + k^2)}.$$

(*b*) Show that the tangents to a parabola at the extremities of a focal chord intersect at right angles on the directrix. 6,6½

3. (*a*) Find the condition that the line $x \cos\alpha + y \sin\alpha = p$ is a normal to the ellipse :

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1.$$

(*b*) From the points on the circle $x^2 + y^2 = a^2$, tangents are drawn to the hyperbola $x^2 - y^2 = a^2$. Show that the locus of the middle points of chord of contact is the curve : 6,6½

$$(x^2 - y^2)^2 = a^2 (x^2 + y^2)^2.$$

Section II

4. (*a*) Evaluate any *two* of the following :

(*i*) $\int_0^{\pi/2} \log \sin x \, dx$

(*ii*) $\int \frac{x^3 + 1}{(x+2)^2 (x-1)} dx$

(*iii*) $\int \frac{x^2 - 1}{1 + x^2} dx$.

(*b*) Evaluate :

$$\int (3x-2)\sqrt{(x^2+x+1)}\,dx\,.$$

7,5½

5. (*a*) Obtain the reduction formula for $\int \sin^n x\,dx$. Hence, evaluate $\int \sin^4 x\,dx$.

(*b*) Evaluate :

$$\int \frac{x+1}{\sqrt{x^2-x+1}}\,dx\,.$$

7,5½

6. (*a*) Find the limit of the sum :

$$\sum_{r=1}^{n-1} \frac{1}{n}\sqrt{\frac{n+r}{n-r}}\,,$$

when n tends to infinity.

(*b*) Find the area of the curvilinear triangle, with one vertex at the origin, lying in the first quadrant and bounded by the curves :

$$y^2 = 4ax, \quad x^2 = 4ay, \quad x^2 + y^2 = 5a^2\,.$$

6,6½

Section III

7. (*a*) Find the perimeter of the cardioid :

$$r = a(1-\cos\theta)\,.$$

(*b*) Find the volume of the solid obtained by revolving the cardioid $r = (1+\cos\theta)$ about the initial line.

6,6½

8. (*a*) Obtain the relation between beta and gamma functions and hence, evaluate the value of $\Gamma\left(\frac{1}{2}\right)$.

(*b*) Evaluate :

$$\Gamma(-1/2), \Gamma(-3/2), \Gamma(-5/2).$$ 7,5½

9. (*a*) Change the order of integration in the double integral :

$$\int_0^{2a} \int_{\sqrt{2ax-x^2}}^{\sqrt{2ax}} f(x, y)\, dxdy.$$

(*b*) Evaluate :

$$\int_0^a \frac{\log(1+ax)}{1+x^2}\, dx.$$ 6,6½