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Roll No. | | | | | | | | | | | |

S. No. of Question Paper : **1857**

Unique Paper Code : **237201** **C**

Name of the Paper : **Probability and Statistical Methods—II (STH-203)**

Name of the Course : **B.Sc. (Hons.) Statistics**

Semester : **II**

Duration : **3 Hours** Maximum Marks : **75**

(Write your Roll No. on the top immediately on receipt of this question paper.)

Attempt *six* questions in all.

Question No. **1** is compulsory.

Attempt **5** more questions by selecting at least *two* questions from each Section.

1\. (*a*) Fill in the blanks :

(*i*) If X and Y are independent binomial variates with parameters $\left(6, \frac{1}{3}\right)$ and $\left(12, \frac{1}{3}\right)$ respectively, then the distribution of X + Y is ....

(*ii*) The distribution for which $\phi(t) = e^{-|t|}$ is ....

(*iii*) If X ~ N(3, 25), then median of the distribution is ....

(*iv*) A random variable X has probability function $f(x) = \frac{1}{2^x}$, $x = 1, 2, 3, \ldots\ldots$, then the Mode of this distribution is ....

(*v*) If V(X) = 2, then V(2 – X) is ....

P.T.O.

(*b*) State the conditions under which Poisson distribution is a limiting case of binomial distribution.

(*c*) Let the p.d.f. of a normal variate be $f(x) = Ce^{-\frac{1}{4}x^2 + x}$. Find C and E(X).

(*d*) Identify the distribution whose m.g.f. is $M_x(t) = 4(3e^{-t} - 1)^{-2}$.

(*e*) The experiment is to toss two balls in 5 boxes in such a way that each ball is equally likely to fall in any box. Let X denotes the number of balls in the fifth box. Find the probability distribution of X.

(*f*) State the relation between first four central moments and cumulants. 1×5,2,2,2,2,2,

Section I

2. (*a*) For the given probability function :

$$f(x) = y_0 x(2 - x);\ 0 \le x \le 2.$$

Find mean, variance, μ_{2n+1}, median mode, β_1 and β_2. Is the distribution symmetric ?

(*b*) If μ_r' is moment of *r*th order about the origin and k_j is the cumulant of *j*th order, prove that : 6,6

$$\frac{\partial \mu_r'}{\partial k_j} = \binom{r-1}{j-1}\mu_{r-j}'.$$

3. (*a*) A continuous r.v. X has the distribution function $F(x) = \begin{cases} 0, & \text{if } x \le 1 \\ k(x-1)^4, & \text{if } 1 < x \le 3 \\ 1, & \text{if } x > 3 \end{cases}$.

Find :

(*i*) k

(*ii*) p.d.f. of x and

(*iii*) $p\left(X < \frac{2}{3} \middle| X > \frac{1}{3}\right)$.

(*b*) The p.d.f. of a random variable X is $f(x) = \frac{1}{6}, -3 \le x \le 3$. Find the p.d.f. of $Y = 4X^2 - 3$ and verify the result. 6,6

4. (*a*) A box contains *a* white and *b* black balls. *c* balls are drawn at random. Find the expected value of the number of white balls drawn.

(*b*) Define the characteristic function of a random variable. Show that the characteristic function of sum of two independent random variables is equal to the product of their characteristic functions.

Justify that, the converse of the above statement need not be true. 6,6

Section II

5. (*a*) If *p* is the constant probability of success at any single trial, obtain the probability of *x* successes out of *n* independent trials. Also find the mode of the resulting distribution.

(*b*) If $X \sim N(\mu, \sigma^2)$, obtain the p.d.f. of $U = \frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2$ and identify the distribution. 6,6

6. (*a*) X is a negative binomial variate with p.m.f. :

$$f(x) = \begin{cases} \binom{k+x-1}{x} q^x p^k, & x = 0, 1, 2, ... \\ 0, & \text{otherwise} \end{cases}$$

Show that the moment recurrence formula is $\mu_{r+1} = q\left(\frac{d\mu_r}{dq} + \frac{rk}{p^2}\mu_{r-1}\right)$ and hence compute μ_2 and μ_3.

(*b*) Find the characteristic function of standard Laplace distribution and hence, find its mean and standard deviation. 6,6

7. (a) Show that for a normal distribution with mean μ and variance σ^2, all odd order central moments vanish and $\mu_{2n} = 1.3.5 \ldots\ldots (2n - 1)\sigma^{2n}$.

(b) If X and Y are two independent random variables each representing the number of failures preceding the first success in a sequence of Bernoulli trials, find the conditional distribution of X given $(X + Y) = n$. 6,6

8. (a) X is a Gamma variate with mean m. Find the moment generating function of $Y = \dfrac{X - m}{\sqrt{m}}$ and show that it approaches $e^{t^2/2}$ as $m \to \infty$. Also interpret the result.

(b) Obtain mean deviation about mean of Poisson distribution with parameter λ. 6,6