Sl. No. of question paper	:	1277	F-4		
Unique paper code	:	2371201			
Name of the Paper	:	Mathematics-I, SDC1-3 (Algebra and Calculus)			
Name of the course	:	B.Sc. (Hons.) Statistics			
Semester	:	II			
Duration	:	03 hours	Maximum marks	:	75 marks

(Write your Roll No. on the top immediately on receipt of this question paper)

Attempt **SIX** questions in all, selecting any **three** from each section.

SECTION-A

1. (a) Express $A = \begin{pmatrix} 0 & 1 & 3 \\ 1 & 2 & 3 \\ 3 & 1 & 1 \end{pmatrix}$ as a product of elementary matrices.

(b) Define **(i)** Nilpotent matrix.

(ii) Unitary matrix.

(iii) Characteristic vector. **(6 ½, 6)**

2. (a) For what values of λ the equations have a solution, and solve them completely:

$$x + y + z = 1$$
$$x + 2y + 4z = \lambda$$
$$x + 4y + 10z = \lambda^2$$

(b) The characteristic roots of a square matrix of order 3 are 1, -1, 2.

Express A^6 as the quadratic polynomial in A. **(6, 6 ½)**

3. (a) Reduce the following quadratic forms to canonical form

$x_1^2 + 4x_2^2 + 4x_3^2 + 4x_1x_2 + 4x_1x_3 + 16x_2x_3$ and hence find its rank, signature and index.

(b) Define orthonormal basis. Describe Schmidt orthogonalization process of converting any set of linearly independent vectors from E^n into an orthonormal basis.

(6 ½, 6)

4. (a) Let $X = \begin{pmatrix} 1 & -1 & -2 \\ 1 & -1 & -2 \\ 1 & -1 & -2 \end{pmatrix}$ find XGX' where G is a generalised inverse of $X'X$.

(b) Find the inverse of matrix $M = \begin{pmatrix} I & Q \\ P & R \end{pmatrix}$ by method of partitioning, where I, the identity sub matrix. **(6, 6 ½)**

SECTION-B

5. (a) Find the n^{th} derivative of $y = \tan^{-1}\frac{x\sin\alpha}{1-x\cos\alpha}$

(b) If $u = \sin^{-1}\left\{\frac{x^{\frac{1}{2}}+y^{\frac{1}{3}}}{x^{\frac{1}{2}}+y^{\frac{1}{2}}}\right\}^{\frac{1}{2}}$ show that

$$x^2\frac{\delta^2u}{\delta x^2} + 2xy\frac{\delta^2u}{\delta x\,\delta y} + y^2\frac{\delta^2u}{\delta y^2} = \frac{\tan u}{144}(13 + tan^2u)$$ **(6 ½, 6)**

6. (a) If $x = r\sin\theta\cos\emptyset$, $y = r\sin\theta\sin\emptyset$, $z = r\cos\theta$ Show that

$$\frac{\partial(x,y,z)}{\partial(r,\theta,\emptyset)} = r^2\sin\theta.$$

(b) Express $\int_0^1 x^m\,(1-x^n)^p\,dx$ in terms of the beta function, and hence evaluate

$\int_0^1 x^5(1-x^3)^{10}\,dx.$ **(5, 7 ½)**

7. (a) Change the order of integration of $\int_0^a \int_0^{\sqrt{a^2-y^2}} \sqrt{a^2-x^2-y^2}\,dy\,dx$ and evaluate it.

(b) Evaluate $\int_0^1 \int_0^1 \frac{dxdy}{\sqrt{(1-x^2)}\sqrt{(1-y^2)}}$ **(6 ½, 6)**

8. Solve the following:

(i) $(x+a)^2\frac{d^2y}{dx^2} - 4(x+a)\frac{dy}{dx} + 6y = x$

(ii) $\frac{d^2y}{dx^2} + 9y = \sec 3x$

(iii) $(6D^2 - D - 2)y = xe^{-x}$ **(4 ½ , 4, 4)**