[This question paper contains 3 printed pages.]

Sr. No. of Question Paper : 2309 **F-4** **Your Roll No...............**

Unique Paper Code : 2371202

Name of the Course : **B.Sc. (H)**

Name of the Paper : Probability & Statistical Methods II [DC–1]

Semester : II

Duration : 3 Hours Maximum Marks : 75

Instructions for Candidates

1. Write your Roll No. on the top immediately on receipt of this question paper.
2. Attempt **fifteen** questions in all, selecting **five** questions from each section.
3. **All** questions carry equal marks.

Section I

1. Prove that if $E(X^r)$ exists, then $E(X^s)$ exists for all $1 \leq s \leq r$.

2. Find moment generating function and cumulant generating function of a random variable whose cumulants are given by $k_r = (r-1)!\, pa^{-r}$; $p > 0$, $a > 0$, $r = 1, 2, \ldots$.

3. Let X be a continuous random variable with distribution function $F(x)$, then prove that

$$E(X) = \int_0^\infty (1 - F(x)dx - \int_{-\infty}^0 F(x)dx$$

provided the integrals exist finitely.

4. Show that if the distribution of a random variable X is symmetrical about zero, then the characteristic function $\phi_X(t)$ is real valued and an even function of t.

P.T.O.

5. A coin is tossed 4 times. Let X denote the number of times a tail is followed immediately by a head. Find the distribution, mean and variance of X.

6. A random variable X assumes the value r with the probability law

$$p\ (X = r) = q^{r-1}\ p,\ r = 1,2,3,\ \dots$$

Find the moment generating function of X and hence its mean and variance.

Section II

7. Obtain mean deviation about mean of binomial distribution with parameters n and p.

8. If X is binomial variate with parameters n and p then prove that $k_{r+1} = pq \left(\frac{dk_r}{dp}\right)$, where k_r is the r^{th} cumulant. Hence find μ_2, μ_3, and μ_4.

9. If X is a Poisson variate with mean λ, then show that $E(X^2)=\lambda E(X+1)$. Also, show that if $\lambda=1$ then $E|X-1|=2/e$.

10. Derive Poisson distribution as a limiting case of negative binomial distribution.

11. What is hypergeometric distribution? Find its mean and variance. How is it related to binomial distribution?

12. Let X and Y be independent random variates such that

$$P(X = r) = P(Y = r) = q^r p,\ r = 0,1,2,3,\ \dots\ and\ p + q = 1.$$

Find the distribution of $X + Y$ and identify its distribution.

Section III

13. Find the mean and mode of gamma distribution with parameter λ. Hence examine the skewness of distribution.

14. Show that for a normal distribution with mean μ and variance σ^2, the central moments satisfy the relation .-

$$\mu_{2n} = (2n-1)\mu_{2n-2}\sigma^2; \mu_{2n+1} = 0$$

Hence. show that $$\mu_{2n} = \frac{(2n)!}{n!}\left(\frac{\sigma^2}{2}\right)^n, n = 1, 2, ...$$

15. Let X be a (standrad) Cauchy variate, find the probability density function of X^2 and identify its distribution.

16. State and prove lack of memory property for exponential distribution.

17. If X and Y are two independent normal variates with common mean μ and variances 4 and 9 respectively. If $P(2X + Y \leq 3) = P(2X - Y \geq 4)$. then find μ.

18. Find the characteristic function of standard Laplace distribution and hence obtain the values of β_1 and β_2.