[This question paper contains 4 printed pages.]

Sr. No. of Question Paper : 1197 **E** **Your Roll No................**

Unique Paper Code : 237253

Name of the Course : **B.Sc. (H) STATISTICS**

Name of the Paper : ALGEBRA II [STH 202]

Semester : II

Duration : 3 Hours Maximum Marks : 75

Instructions for Candidates

1. Write your Roll No. on the top immediately on receipt of this question paper.
2. Attempt **SIX** questions in all.
3. Question No. **1** is compulsory.
4. Attempt **Five** more questions selecting atleast **two** questions from each Section.
5. Use of simple calculator to be allowed.
6. Attempt all parts of a question in continuation.

1. (a) State whether the following statements are true or false :

   (i) Inverse of $E_{ij}(k) \neq 0$ is $E_{ij}\left(\frac{1}{k}\right)$.

   (ii) Square matrix is non-singular if zero is not one of its characteristic roots.

   (iii) Generalized inverse of a matrix of any order always exists.

   (iv) The equation AX = 0 has a non-zero solution-if and only if the rank 'r' of A is less than the number 'n' of its columns i.e. of the unknowns.

   (v) The set of all positive integers is a ring for ordinary addition and multiplication.

   (b) Let $A = \begin{pmatrix} 5 & 3 & 1 \\ 5 & 3 & 1 \\ 5 & 3 & 1 \end{pmatrix}$, without any calculation find one eigen value of A. Justify your answer.

(c) Is the matrix $\begin{pmatrix} 0 & 1 & 3 \\ 1 & 2 & 3 \\ 3 & 1 & 1 \end{pmatrix}$ equivalent to I_3 ? Justify.

(d) Define :

(i) Index of a real quadratic form;

(ii) Discriminant of the quadratic form.

(e) Show that no skew symmetric matrix can be of rank 1.

(f) If A is a $m \times n$ matrix of rank p and L and B are two non-singular matrices of order m and n respectively, then write down the form of LAB.

(5,2,2,2,2,2)

SECTION I

2. (a) Prove that every non-singular matrix can be reduced to the normal form by :

(i) E-row transformation only; and

(ii) E-column transformation only.

Hence or otherwise find the rank of the matrix :

$$A = \begin{pmatrix} 2 & 3 & -1 & -1 \\ 1 & -1 & -2 & -4 \\ 3 & 1 & 3 & -2 \\ 6 & 3 & 0 & -7 \end{pmatrix}$$

(b) Discuss for all values of k, the solutions for the following system of equations :

$2x + 3ky + (3k + 4)z = 0$

$x + (k + 4)y + (4k + 2)z = 0$

$x + 2(k+ 1)y + (3k + 4)z = 0$ (6,6)

3. (a) If X_i and X_j are the characteristic vectors corresponding to two distinct characteristic roots λ_i and λ_j respectively of a $n \times n$ square matrix A then

(i) X_i and X_j are always linearly independent.

(ii) X_i and X_j are orthogonal if A is symmetric.

(b) Show that the matrix $A = \begin{bmatrix} 1 & 2 & 0 \\ 2 & -1 & 0 \\ 0 & 0 & -1 \end{bmatrix}$ satisfies Cayley-Hamilton theorem.

Hence or otherwise obtain the value of A^{-1} and A^{-2}. (6,6)

4. (a) Identify the nature of the quadratic form $4x^2 + 9y^2 + 2z^2 + 8yz + 6zx + 6xy$ and hence find the rank, index and signature of the form.

(b) Prove that the definiteness of a quadratic form is invariant under non-singular linear transformation. (7,5)

5. (a) If $B = \begin{bmatrix} 2 & \sqrt{2} \\ \sqrt{2} & 1 \end{bmatrix}$, then find the characteristic equation of B and verify that the matrix B satisfies the equation. Also find the characteristic roots and corresponding characteristic vectors of B.

(b) Prove that the characteristic roots of a square matrix A of order 3 are same as that of any of its transformed matrix, PAP^{-1} where P is any non-singular matrix of order 3. Also if

$$P = \begin{pmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{pmatrix}; \; A = \frac{1}{2}\begin{pmatrix} b+c & c-a & b-a \\ c-b & c+a & a-b \\ b-c & a-c & a+b \end{pmatrix}.$$

Determine the characteristic roots of the matrix A. (6,6)

SECTION II

6. (a) If G is a generalized inverse of X′X, then prove that

(i) G′ is also a generalized inverse of X′X.

(ii) XGX′X = X, ie. GX′ is a generalized inverse of X.

(iii) XGX′ is invariant to G.

(iv) XGX′ is symmetric whether G is or not.

(b) Discuss the algorithm for finding a generalized inverse of a given matrix.

Hence, compute a generalized inverse of the matrix $\begin{pmatrix} 18 & 2 & 46 \\ 2 & 1 & 2 \\ 46 & 2 & 130 \end{pmatrix}$. (6,6)

7. (a) If $A = \begin{pmatrix} \alpha & \beta \\ \gamma & \delta \end{pmatrix}$, where α, β, γ and δ are sub-matrices and $|\alpha| \neq 0$, then find the inverse of A by method of partitioning.

(b) Define a commutative group and order of a finite group. Prove that the totality of all positive rational numbers forms an abelian group under the composition defined by $a * b = (ab) / 2$. (6,6)

8. (a) Define orthonormal basis. Using the Schmidt Orthogonalisation process construct an orthonormal basis for E^3 from the following set of basis vectors :

$$a_1 = [1,1,1];\ a_2 = [0,1,1];\ a_3 = [0,0,1]$$

(b) (i) Do the vectors (1,1,0), (0,1,2) and (0,0,1) form a basis of $V_3(R)$?

(ii) Express the vector $v = (3,1,-4)$ as a linear combination of the vectors $V_1 = (1,1,1)$, $V_2 = (0,1,1)$ and $V_3 = (0,0,1)$. (6,3,3)