This question paper contains 4 printed pages]

Roll No. | | | | | | | | | | | |

S. No. of Question Paper : **1196**

Unique Paper Code : **237252** **E**

Name of the Paper : **Calculus II**

Name of the Course : **B.Sc. (H) Statistics**

Semester : **II**

Duration : **3 Hours** Maximum Marks : **75**

(Write your Roll No. on the top immediately on receipt of this question paper.)

Attempt *six* questions in all, selecting at least *two* questions from each Section.

All questions carry equal marks.

Attempt *all* the parts of a question in continuation.

Section I

1. (*a*) Prove that the general equation :

$$ax^2 + 2hxy + by^2 + 2gx + 2fy + c = 0$$

represents two parallel straight lines if $h^2 = ab$ and $bg^2 = af^2$. Also prove that the distance between them is :

$$2\sqrt{\frac{g^2 - ac}{a(a+b)}}.$$

(*b*) Show that the straight lines given by the equation :

$$(ax + by)^2 - 3(bx - ay)^2 = 0$$

form with the line $ax + by + c = 0$, an equilateral triangle whose area is :

$$\frac{c^2}{\sqrt{3}(a^2 + b^2)}.$$

2. (*a*) Show that the equation of the straight line meeting the circle $x^2 + y^2 = a^2$ at two points, each at equal distance d from a point (x_1, y_1) on the circumference, is :

$$xx_1 + yy_1 - a^2 + \frac{1}{2}d^2 = 0.$$

(*b*) If e_1 and e_2 are the eccentricities of a hyperbola and its conjugate, then prove that :

$$\frac{1}{e_1^2} + \frac{1}{e_2^2} = 1.$$

3. (*a*) Prove that the chord :

$$y - x\sqrt{2} + 4a\sqrt{2} = 0$$

of the parabola $y^2 = 4ax$ is a normal to the curve and its length is $6\sqrt{3}a$.

(*b*) A line through the origin meets the circle $x^2 + y^2 = a^2$ at point P and the hyperbola $x^2 - y^2 = a^2$ at Q. Prove that the locus of the point of intersection of the tangent at P to the circle with the tangent at Q to the hyperbola is the curve :

$$\left(a^4 + 4y^4\right)x^2 = a^6.$$

Section II

4. (*a*) Evaluate any *two* of the following :

(*i*) $\displaystyle\int \frac{x^3 \sin^{-1} x}{\sqrt{1 - x^2}}\, dx$

(*ii*) $\displaystyle\int \frac{2x^2 + 3}{\sqrt{3 - 2x - x^2}}\, dx$

(*iii*) $\displaystyle\int \frac{x\left(2x^2 - x + 5\right)}{\left(x^2 + 2x + 2\right)^2}\, dx$

(*iv*) $\displaystyle\int \frac{x^2 + 9}{x^4 + 7x^2 + 81}\, dx.$

(*b*) Obtain the reduction formula for :

$$\int \frac{dx}{\left(x^2 + a^2\right)^n}$$

and hence evaluate :

$$\int_0^\infty \frac{dx}{\left(x^2 + 4\right)^5}.$$

5. (*a*) Find the limit, when n tends to infinity, of the sum :

$$\sum_{r=1}^{n-1} \frac{1}{n}\sqrt{\frac{n+r}{n-r}}.$$

(*b*) Find the area of the region included between the curves $x^2 + y^2 = 2ax$ and $y^2 = ax$.

6. (*a*) Find the length of any arc of the curve $r = ae^{\theta \cot \alpha}$ taking $s = 0$ when $\theta = 0$; s is length of the arc.

(*b*) Show that the volume of the solid generated by revolving the curve :

$$x^{\frac{2}{3}} + y^{\frac{2}{3}} = a^{\frac{2}{3}}$$

about x-axis is $\dfrac{32\pi a^3}{105}$.

7. (*a*) Show that :

$$\int_0^1 \frac{x^{m-1}(1-x)^{n-1}}{(a+bx)^{m+n}}\, dx = \frac{B(m, n)}{(a+b)^m a^n}.$$

(*b*) Prove that :

$$\int_0^1 x^m (\log x)^n \, dx = \frac{(-1)^n n!}{(m+1)^{n+1}},$$

$m > -1$ honds for all positive integers n.

8. (*a*) Evaluate :

$$\int_0^{\pi} \int_0^{a(1+\cos\theta)} r dr \, d\theta.$$

(*b*) Change the order of integration and hence evaluate the double integral :

$$\int_0^1 \int_{4y}^4 e^{x^2} dx \, dy.$$