[This question paper contains 4 printed pages.]

Sr. No. of Question Paper : 1198 **E** **Your Roll No...............**

Unique Paper Code : 237201

Name of the Course : **B.Sc. (Hons.) Statistics**

Name of the Paper : Probability and Statistical Methods – II [STH–203]

Semester : II

Duration : 3 Hours Maximum Marks : 75

Instructions for Candidates

1. Write your Roll No. on the top immediately on receipt of this question paper.
2. Attempt Question No. 1 which is compulsory and any five of the remaining question selecting at least two questions from each Section.
3. Attempt all parts of a question in continuation.

1. (a) Let X be a continuous random variable with c.d.f. :

$$F(x) = \begin{cases} 0 & ; x \le 0 \\ k(1-e^{-x})^2 & ; x > 0, \end{cases}$$

find :

(i) k,

(ii) P(X > 2).

(b) Let X be a random variable with probability distribution :

x	–1	1	3	5
P(X=x)	1/6	1/6	1/6	1/2

Evaluate $E(2X + 1)^2$.

(c) State the relationship between the following :

(i) $E(X^r)$ and $M_x(t)$,

(ii) $\varphi_x(t)$ and p.d.f. of X,

(iii) Mean and variance of a negative binomial distribution.

(d) Let $X \sim B(2,p)$ and $Y \sim B(4, p)$.
If $P(X \geq 1) = 5/9$, then find $P(Y \geq 1)$.

(e) Let X_1, X_2 and X_3 be i.i.d. random variables having the same distribution given by

$$f(x) = \frac{1}{\sqrt{8\pi}} e^{-\frac{(x-1)^2}{8}}; \quad -\infty < x < \infty,$$

find the distribution of :

(i) $X_1 - 2X_2 + X_3$, (ii) $\sum_{i=1}^{3} \frac{X_i}{3}$

(f) Fill in the blanks :

(i) $E|X - A|$ is minimum when A is ....

(ii) If $var(X) = 1$, then $var(2X + 3) = ...$

(iii) If X_1 and X_2 are independent random variables with characteristic functions $\varphi_{X_1}(t)$ and $\varphi_{X_2}(t)$ respectively, then $\varphi_{X_1-X_2}(t) = ...$

(g) Name a probability distribution which is symmetric but not normal.

(2,2,3,2,2,3,1)

SECTION I

2. (a) Let X be a random variable with p.d.f. :

$$f(x) = \frac{2a}{\pi}\left(\frac{1}{a^2 + x^2}\right); \quad -a \leq x \leq a.$$

Show that the distribution is symmetrical

and $\mu_2 = \frac{a^2(4-\pi)}{\pi}, \mu_4 = a^4\left(1 - \frac{8}{3\pi}\right)$.

(b) An urn contains balls numbered 1, 2, 3. First a ball is drawn from the urn and then a fair coin is tossed as many number of times as shown on the drawn ball. Find the expected number of heads. (6,6)

3. (a) Define expectation. State and prove multiplication theorem of expectation.

(b) Let p(x) be the probability function of a discrete random variable X which assume the values –3, –1, 2 and 4 such that $2P(X = -3) = 3P(X = -1) = P(X = 2) = 5P(X = 4)$. Find the probability mass function and c.d.f. of the random variable X. Also draw the graph of c.d.f. (6,6)

4. (a) Let the p.d.f. of random variable X be :

$$f(x) = \frac{1}{6}, \quad -3 \le x \le 3.$$

Find the p.d.f. of $Y = 2X^2 - 3$.

(b) If X is a random variable having cumulants k_r; r = 1, 2, ... given by

$$k_r = (r-1)!\ \lambda^{-r},\ \lambda > 0,$$

then find the characteristic function of X and identify the distribution. (6,6)

5. (a) The c.d.f. of a random variable X is :

$$F(x) = \begin{cases} 0 & ; -\infty \le x < 0 \\ \dfrac{x^2}{4} & ; 0 \le x < 1 \\ \dfrac{2x-1}{4} & ; 1 \le x < 2 \\ \dfrac{-x^2}{4} + \dfrac{3x}{2} - \dfrac{5}{4} & ; 2 \le x < 3 \\ 1 & ; 3 \le x < \infty \end{cases}$$

Find the probability function (p.m.f./p.d.f.) of the random variable X.

Also find (i) $P(-2 \le X < 2)$, (ii) $P(X > 2.5)$.

(b) Obtain the m.g.f. of a random variable X having p.d.f.

$$f(x) = \begin{cases} x & ; 0 \leq x < 1 \\ 2 - x & ; 1 \leq x \leq 2 \\ 0 & ; \text{elsewhere} \end{cases}$$

Determine first four moments about mean. (6,6)

SECTION II

6. (a) Find the mode of a binomial distribution with parameter n and p. Hence find mode of $X \sim B(8,1/3)$.

(b) If X is a Poisson variate with mean λ, show that $E(X^2) = \lambda E(X+1)$.
Also show that if $\lambda = 1$, $E|X - 1| = 2/e$. (6,6)

7. (a) If X is a negative binomial variate with p.m.f. given by

$$p(x) = \binom{k+x-1}{x} q^x p^k, \quad x = 0,1,2,...$$

Show that the recurrence relation for moments is :

$$\mu_{r+1} = q\left(\frac{d\mu_r}{dq} + \frac{rk}{p^2}\mu_{r-1}\right), \quad r = 1,2,3,...$$

Hence find Var(X).

(b) Find mean deviation about mean of a two parameter Laplace distribution. (6,6)

8. (a) If $X \sim$ Cauchy distribution (λ, μ), then show that
$1/X \sim$ Cauchy distribution $(\lambda/(\lambda^2 + \mu^2), \mu/(\lambda^2 + \mu^2))$.

(b) Write the equation of a normal probability curve with mean μ and standard deviation σ. State eight main characteristics of the normal probability curve. (6,6)