[This question paper contains 4 printed pages.]

1048 *Your Roll No.* ....

B.Sc. (Hons.) / II **C**

STATISTICS – Paper IX

B–221 : (Mathematics – IV)

(Admissions of 1999 and onwards)

Time : 2 Hours *Maximum Marks : 38*

(Write your Roll No. on the top immediately on receipt of this question paper.)

Answer **four** *questions in all, selecting any* **two** *questions from each section.*

SECTION I

1. (a) Prove that when G is a generalised inverse of $X'X$, then

   (i) G' is also a generalised inverse of $X'X$.

   (ii) $XGX'X = X$ i.e. GX' is a generalised inverse of X.

   (iii) XGX' is invariant to G.

   (iv) XGX' is symmetric, whether G is symmetric or not.

P.T.O.

(b) Show that elementary matrices are non-singular and obtain their inverses. (5,4½)

2. (a) Matrices A and B are partitioned as shown below :

$$A = \left(\begin{array}{cc|cc} 3 & 2 & 1 & 4 \\ 4 & 6 & 5 & 0 \\ \hline 7 & 1 & 0 & 2 \end{array}\right) \qquad B = \left(\begin{array}{cc} 1 & 7 \\ 0 & 6 \\ \hline 1 & 2 \\ 5 & 2 \end{array}\right)$$

Prove by direct multiplication and by block multiplication that the same result AB is obtained either way.

(b) Investigate for what values of μ, the system of equations :

$$\mu x + y + z = 1$$

$$x + \mu y + z = \mu$$

$$x + y + \mu z = \mu^2$$

has a unique solution and also state the nature of solution when $\mu = 1$ and $\mu = -2$. (4½,5)

3. (a) Find the characteristic roots and characteristic vectors of the following matrix :

$$A = \begin{pmatrix} 1 & 2 & 3 \\ 0 & 2 & 3 \\ 0 & 0 & 2 \end{pmatrix}$$

(b) If S is a real skew-symmetric matrix of order n and I is a unit matrix, then prove that

(i) $(I+S)$ is a non singular matrix

(ii) $A = (I - S)(I + S)^{-1}$ is an orthogonal matrix of order n. (5½,4)

SECTION II

4. (a) Reduce the following matrix to the diagonal form and interpret the result in terms of quadratic forms

$$A = \begin{pmatrix} 1 & 1 & -1 \\ 1 & 2 & 1 \\ -1 & 1 & 3 \end{pmatrix}$$

(b) Define ring with zero divisors and integral domain. Prove that the set of all the n, nth roots of unity forms a finite abelian group of order n, with respect to multiplication. (4½,5)

5. (a) Express (1,2,3) as a linear combination of (1,1,1), (2,−1,1) and (1,−2,5) in $V_3(R)$.

(b) Using Schmidt orthogonalization procedure, construct an orthonormal basis from the set of vectors (2,3,0), (6,1,0) and (0,2,4).

(c) Show that the set of vectors $\{(0,1,0), (1,0,1), (1,1,0)\}$ constitutes a basis of the real vector space R^3. (3,3½,3)

6. (a) Find the value of

$$\Gamma\left[1/n\right]\ \Gamma\left[2/n\right]\ \Gamma\left[3/n\right]\ \ldots\ldots\ \Gamma\left[(n-1)/n\right]$$

(b) Change the order of integration in

$$\int_{2}^{4}\int_{4/x}^{(20-4x)/(8-x)} (4-y)\,dy\,dx$$

and evaluate. (5,4½)