[This question paper contains 4 printed pages.]

1419 *Your Roll No.* ....

B.Sc. (Hons.)/II **A**

STATISTICS – Paper IX

(Mathematics – IV)

(For Admissions of 1999 and onwards)

Time : 2 Hours *Maximum Marks : 38*

(Write your Roll No. on the top immediately on receipt of this question paper.)

Attempt **Four** *questions in all, selecting* **two** *questions from each Section.*

SECTION I

1. (a) Show that every non-singular matrix can be expressed as the product of elementary matrices. Hence express

$$A = \begin{pmatrix} 1 & 3 & 3 \\ 1 & 4 & 3 \\ 1 & 3 & 4 \end{pmatrix}$$

as a product of elementary matrices.

(b) Prove that consistent equations $Ax = y$ have a solution $x = Gy$ if and only if $AGA = A$, where G is a generalised inverse of the matrix A.

(6½,3)

P.T.O.

2. (a) For what values of K does the system of equations

$$x + y + z = 1$$
$$x + 3y + 9z = K$$
$$x + 9y + 33z = K^2$$

have a solution ?

(b) Given the following matrices

$$A = \begin{bmatrix} 2 & 1 \\ 3 & 4 \end{bmatrix}, \ B = \begin{bmatrix} 1 & 7 & 2 \\ 0 & 6 & 5 \end{bmatrix},$$

show by direct multiplication that $AB = (Ab_1, Ab_2, Ab_3)$ where B has been partitioned into the column vectors b_i, $i = 1, 2, 3$. (6,3½)

3. (a) If $A = (a_{ij})$ is a square matrix of order n, then show that

(i) the sum of its characteristic roots is equal to tr(A)

(ii) the product of its characteristic roots is equal to $|A|$.

(b) Obtain the characteristic equation of the matrix

$$A = \begin{bmatrix} 1 & 0 & 2 \\ 0 & 2 & 1 \\ 2 & 0 & 3 \end{bmatrix}$$

Hence calculate its inverse. (5,4½)

SECTION II

4. (a) Find the rank and index of the real quadratic form

$$2x_1^2 + x_2^2 - 3x_3^2 - 8x_2x_3 - 4x_3x_1 + 12x_1x_2.$$

(b) Prove that if G is an abelian group, then for all a, b, $\in$ G and for any integer n, $(a.b)^n = a^n b^n$.

(5,4½)

5. (a) If α, β, γ are linearly independent vectors of V(C), where C is the field of complex numbers, then so are $\alpha + \beta$, $\beta + \gamma$ and $\gamma + \alpha$.

(b) Define an orthogonal basis. Given the basis vectors [1, 0, 0], [0, 1, 1] and [0, 0, 1] for E^3, find which vector can be removed from the basis and be replaced by $\underset{\sim}{b} = [4, 3, 6]$ while still maintaining a basis ?

(c) Using Schmidt orthogonalization process, construct an orthonormal basis for E^3 from the following set of basis vectors :

$$\underset{\sim}{a}_1 = [2, 6, 3], \ \underset{\sim}{a}_2 = [9, 1, 0], \ \underset{\sim}{a}_3 = [1, 2, 7]$$ (3,3,3½)

6. (a) Show that

(i) $$\Gamma(m)\,\Gamma\left(m + \frac{1}{2}\right) = \frac{\sqrt{\pi}}{2^{2m-1}}\,\Gamma(2m)$$

(ii) $\int_0^\infty \int_0^\infty e^{-(ax^2+by^2)} x^{2m-1} y^{2n-1} \, dx \, dy = \frac{\overline{|m}\,\overline{|n}}{4a^m b^n}$

(b) Change the order of integration in

$$\int_0^a \int_0^x \frac{f'(y)\,dy\,dx}{\sqrt{(a-x)(x-y)}}.$$

Hence find its value. (5,4½)