[This question paper contains 4 printed pages.]

1420 *Your Roll No.* ....

B.Sc. (Hons.) / II **A**

STATISTICS – Paper X

(Mathematics – V)

(For Admissions of 1999 and onwards)

Time : 2 Hours *Maximum Marks : 38*

(Write your Roll No. on the top immediately on receipt of this question paper.)

Attempt **four** *questions in all, selecting* **two** *questions from each Section.*

SECTION I

1. (a) Define infimum of a non-empty bounded set S of real numbers. Prove that a real number m is the infimum of S if and only if

   (i) $x \geq m \quad \forall \; x \in S$ and

   (ii) For each $\varepsilon > 0$, $\exists$ a real number $y \in S$ such that $y < m + \varepsilon$.

   (b) (i) Give an example of a set which is not a neighbourhood of exactly one of its points.

(ii) Find the supremum and infimum of the set $\{x \in R, : |x-1| < 2\}$.

(iii) Give an example of an infinite closed set which is not an interval.

(iv) If $I_n = \left[\frac{1}{n}, 2\right]$, $x \in N$, then find $\bigcup_{n \in N} I_n$.

(v) What is the derived set of rational numbers ?

(4½,5)

2. (a) Define the convergence of a sequence. Prove that a convergent sequence

(i) is bounded and

(ii) has a unique limit

(b) Show that the sequence $<a_n>$, where

$$a_1 = 1,$$

$$a_{n+1} = \sqrt{3a_n}, \; n \geq 1$$

converges to 3. (5½,4)

3. (a) Let $\sum_{n=1}^{\infty} u_n$ be a positive term series such that

$$\lim_{n\to\infty} \frac{u_n}{u_{n+1}} = \ell \text{ where } \ell > 1.$$

Prove that $\sum_{n=1}^{\infty} u_n$ converges. Discuss the case when $\ell = 1$.

(b) Examine the convergence of

(i) $\sum_{n=1}^{\infty} \frac{n}{n+1}$

(ii) $\operatorname{Sin} 1 + \operatorname{Sin}\frac{1}{2} + \operatorname{Sin}\frac{1}{3} + \cdots$ (5½,4)

SECTION II

4. (a) Examine the continuity and derivability of the function

$$f(x) = |x| + |x-1|, \; x \in R$$

at $x = 0$ and $x = 1$.

(b) State Lagrange's Mean value theorem. Use it to show that

$$\frac{v-u}{1+v^2} < \tan^{-1} v = \tan^{-1} u < \frac{v-u}{1+u^2},$$

where $0 < u < v$. (5½,4)

5. (a) Obtain Maclaurin's series expansion of e^x, $x \in R$. Hence or otherwise show that

$$e^2 - 3 = \frac{2^2}{2!} + \frac{2^3}{3!} + \frac{2^4}{4!} + \cdots$$

(b) Evaluate

(i) $\lim_{x \to 0} \frac{\log(1+x) - x}{1 - \cos x}$

(ii) $\lim_{x \to 1} (2-x)^{\tan\left(\frac{\pi x}{2}\right)}$ (5½,4)

P.T.O.

6. (a) Show that

$$\int_0^\infty e^{-x} x^{n-1} dx \text{ is convergent iff } n > 0.$$

(b) Show that the sequence $\{f_n\}$ of functions, where

$$f_n(x) = \frac{1}{(x+n)},$$

is uniformly convergent in [0, a], where $a > 0$.

(5½,4)