[This question paper contains 3 printed pages.]

1421 *Your Roll No.* ....

B.Sc. (Hons.) / II **A**

STATISTICS – Paper XI

(Mathematics – VI)

(Admissions of 1999 and onwards)

Time : 2 Hours *Maximum Marks : 38*

(Write your Roll No. on the top immediately on receipt of this question paper.)

Attempt **four** *questions in all, selecting at least* **two** *questions from each Section.*

SECTION I

1. (a) Prove that

$$\frac{\Delta^n 0^m}{n!} = \frac{n\, \Delta^n 0^{m-1}}{\lfloor n} + \frac{\Delta^{n-1} 0^{m-1}}{(n-1)!},$$

where n and m are positive integers.

(b) Find the sum to n terms of the series

$$1.2\, \Delta x^n - 2.3\, \Delta^2 x^n + 3.4\, \Delta^3 x^n - 4.5\, \Delta^4 x^n + \ldots$$

(5½,4)

2. (a) Obtain Lagrange's interpolation formula in the form

P.T.O.

$$f(x)=\sum_{r=1}^{n}\frac{L(x)f(x_r)}{(x-x_r)L'(x_r)}=\sum_{r=1}^{n}L_r(x)\,f(x_r),\text{ where}$$

$$L(x) = (x-x_1)(x-x_2)\text{ - - - }(x-x_n).$$

(b) The values of f(x) are given at a, b and c. Show that under certain conditions, the maximum or minimum is attained at

$$x = \sum(b^2-c^2)f(a)/\left[2\sum(b-c)f(a)\right]$$ (5½,4)

3. (a) If δ is the operator with usual meaning and if $hD \equiv U$, where h is the interval of differencing, prove

$$\frac{U}{\delta}=\frac{2}{\delta}\sinh^{-1}\frac{\delta}{2}=1-\frac{\delta^2}{24}+\frac{3}{640}\delta^4-\frac{5}{7168}\delta^6+0(\delta^8)$$

(b) Derive Gauss's forward formula for equal intervals. (5½,4)

SECTION II

4. (a) Show that the Cote's numbers are symmetric and derive Trapezoidal rule using Cote's formula.

(b) If f(x) is a polynomial in x of third degree and

$$u_{-1}=\int_{-3}^{-1}f(x)dx,\quad u_0=\int_{-1}^{1}f(x)dx,\quad u_1=\int_{1}^{3}f(x)dx,$$

then show that $f(0)=\frac{1}{2}\left(u_0-\frac{\Delta^2 u_{-1}}{24}\right)$. (5½,4)

5. (a) Using a suitable result, evaluate the value of $\sum_{0}^{\infty} a^x f(x)$ and hence find the sum to n terms of the following series :

$2.2 + 7.4 + 14.8 + 23.16 + 34.32 + \cdots$

(b) Find the sum of the following series to n terms

12, 40, 90, 168, 280, 432, ... (5½,4)

6. Solve any **three** of the following difference equations :

(i) $u_{x+1} - pa^{2x} u_x = qa^{x^2}$

(ii) $u_{x+1} u_x - a^x(u_{x+1} - u_x) - 1 = 0$

(iii) $u_{x+2} - 7u_{x+1} - 8u_x = x^{(2)} 2^x$

(iv) $u_{x+2} - 4u_{x+1} + 4u_x = 2^x$ (9½)