[This question paper contains 4 printed pages.]

1423 *Your Roll No.* ....

B.Sc. (Hons.) / II **A**

STATISTICS – PAPER-XIII

(Statistical Methods – II)

(Admissions of 1999 and onwards)

Time : 2 Hours *Maximum Marks : 38*

(Write your Roll No. on the top immediately on receipt of this question paper.)

Attempt **Four** *questions in all, Selecting* **two** *questions from each Section.*

SECTION I

1. (a) Find the minimum sample size n for estimating population proportion P with confidence coefficient $(1-\alpha)$ and permissible error E in estimate.

   (b) What is meant by a statistical hypothesis ? What are the two types of errors of decision that arise in testing a hypothesis ?

   (c) Find the m.g.f. of $Z = \log \chi^2$ where $\chi^2 \sim \chi^2_n$. If χ^2_1 and χ^2_2 are independent χ^2–variates each with n d.f. and $u = \frac{\chi_1^2}{\chi_2^2}$, deduce that for positive integer K

$$E(U^K) = \frac{\Gamma\left(\frac{n}{2}+K\right)\Gamma\left(\frac{n}{2}-K\right)}{\left[\Gamma\left(\frac{n}{2}\right)\right]^2} \qquad (3,2\tfrac{1}{2},4)$$

2. (a) If $X_1 + X_2 - - - X_n$ is a random sample from $N(\mu, \sigma^2)$, find the p.d.f., mean and variance of

$$S = \left[\sum_{i=1}^{n} (X_i - \bar{X})^2 / (n-1)\right]^{\frac{1}{2}}$$

(b) Let X_1, X_2 be independent random variables following the probability law $f(x) = e^{-x}; x > 0$.

Find the distribution of $\frac{X_1}{X_2}$ and identify it.

(3½,4)

3. (a) Show that t-distribution is symmetrical about its mean.

(b) p_1, p_2 are sample proportions with respect to prevalence of a certain attribute, being based on random samples from two different populations with population proportion P_1 and P_2. Let p be the pooled estimate of P_1 as well as P_2. Then show that $var(p) = cov(p_1, p)$ under the assumption $P_1 = P_2$.

(c) If X is a χ^2–variate with n d.f., then for large n find the distribution of $\sqrt{2X}$ and identify it.

(3,3,3½)

SECTION II

4. (a) If $\bar{X}$ and S^2 are the usual sample mean and sample variance (unbiased estimate of σ^2) based on a random sample of n observations from $N(\mu, \sigma^2)$ and if $T = \frac{(\bar{X} - \mu)\sqrt{n}}{S}$, prove that

$$\text{Var}(T) = \frac{n-1}{n-3} \text{ and } \text{cov}(\bar{X}, T) = \frac{\sigma\sqrt{n-1}\ \Gamma[(n-2)/2]}{\sqrt{2n}\ \Gamma\left(\frac{n-1}{2}\right)}$$

(b) Obtain the mode of F-distribution with (n_1, n_2) d.f. and show that it lies between 0 and 1. (5½,4)

5. (a) If X is a random variable following Poisson distribution with parameter λ, which itself is a random variable such that $2\alpha\lambda$ $(\alpha > 0)$, is a χ^2-variate with 2p degrees of freedom, obtain the unconditional distribution of X and identify it.

(b) Suppose $X_1, X_2, ---, X_n$ $(n \geq 2)$ are independent variates each distributed as $N(0, \sigma^2)$. If

$$u = \frac{X_1}{\sqrt{\frac{1}{n}\cdot\sum_{i=1}^{n} X_i^2}} \quad \text{and} \quad V = \sqrt{\frac{n-1}{n}}\,\frac{V}{\sqrt{1-\frac{V^2}{n}}},$$

show that $V \sim t_{n-1}$ distribution. (5,4½)

6. Write short notes on any **three** of the following :

(i) Linear orthogonal transformation

(ii) Logarithmic Transformation

(iii) Fisher's Z-distribution

(iv) Test of difference of means when population variances are equal and known (9½)