[This question paper contains 4 printed pages.]

**1049** *Your Roll No.* .............

**B.Sc. (Hons.) / II** **C**

STATISTICS – Paper X

B–222 : (Mathematics – V)

(For Admissions of 1999 and onwards)

*Time : 2 Hours* *Maximum Marks : 38*

*(Write your Roll No. on the top immediately on receipt of this question paper.)*

*Attempt* **four** *questions in all, selecting* **two** *questions from each Section.*

## SECTION I

1. (a) (i) State Bolzano Weierstrass Theorem for sets.

(ii) Give an example of an open set which is not an interval.

(iii) Is the set $[0,1] \cup [2,3]$ closed ?

(b) Define infimum of a set. Let S be a non empty set of real numbers which is bounded below. Prove

P.T.O.

that a real number $t$ is the infimum of S iff the following conditions hold :

(i) $x \geq t \quad \forall x \in S$,

(ii) For each positive real number $\varepsilon$, there is a real number $x \in S$ such that $x < t + \varepsilon$.

(4½,5)

2. (a) Use Cauchy Convergence Criteria to show the sequence $<a_n>$ where

$$a_n = 1 + \frac{1}{2} + \frac{1}{3} + ... + \frac{1}{n}, \qquad n \in N$$

doesn't converge.

(b) Define a monotonic sequence. Prove that, a monotonically increasing sequence which is bounded above, converges. (4½,5)

3. (a) If $\sum u_r$ is a positive term series, such that

$\lim_{n\to\infty} \frac{u_n}{u_{n+1}} = l$, then show that the series diverges if $l < 1$.

(b) Examine the convergence of the following series :

(i) $\frac{1}{2}+\frac{2}{2^2}+\frac{3}{2^3}+\frac{4}{2^4}+\ldots$

(ii) $1-\frac{1}{3}+\frac{1}{5}-\frac{1}{7}+\ldots$ (4½,5)

## SECTION II

4. (a) Show that the function

$f(x) = [x], \quad x \in [0\ 3[$ is discontinuous at $x = 1$ and $x = 2$.

(b) State Lagrange's mean value theorem. Find a point $c \in ]0\ \frac{1}{2}[$ of Lagrange's mean value theorem if $f(x) = x(x-1)(x-2), \quad x \in \left[0\ \frac{1}{2}\right]$. (4½,5)

5. (a) Obtain Maclaurin's series expansion of cos x, $x \in R$.

(b) Evaluate

(i) $\lim_{x\to 0} \frac{1-\cos x}{x^2}$,

(ii) $\lim_{x\to 0+} x^x$. (4½,5)

6. (a) State and prove Weierstrass' s M-test for uniform convergence of a series.

(b) Examine the convergence of $\int_2^\infty \frac{dx}{(x-1)^3}$.

(5,4½)