[This question paper contains 4 printed pages.]

1050 *Your Roll No.* ....

B.Sc. (Hons.) / II **C**

STATISTICS – Paper XI

B–223 : (Mathematics – VI)

(Admissions of 1999 and onwards)

Time : 2 Hours *Maximum Marks : 38*

(Write your Roll No. on the top immediately on receipt of this question paper.)

Attempt **four** *questions in all, selecting at least* **two** *questions from each Section.*

SECTION A

1. (a) Show that Newton-Gregory formula can be written as

$$P_n(x) = f(0) + x\Delta f(0) - xa_1\Delta^2 f(0) + xa_1a_2\Delta^3 f(0) - \ldots,$$

where $a_1 = 1 - \frac{1}{2}(x+1)$, $a_2 = 1 - \frac{1}{3}(x+1)$, etc.

P.T.O.

(b) Find the relation between α, β, γ in order that $\alpha + \beta x + \gamma x^2$ may be expressible in one term in the factorial notation. (5,4½)

2. (a) Obtain Lagrange's interpolation formula in the form

$$f(x) = \sum_{r=1}^{n} \frac{L(x)f(x_r)}{(x-x_r)L'(x_r)} = \sum_{r=1}^{n} L_r(x)f(x_r),$$

where $L(x) = (x - x_1)(x - x_2) \ldots (x - x_n)$.

(b) Obtain Newton's divided difference formula. Hence obtain Newton's forward interpolation formula. (5,4½)

3. (a) Derive Gauss's forward formula for equal intervals.

(b) If D and ∇ are the operators with usual meanings and if $hD \equiv U$, where h is the interval of differencing, prove that

(i) $\nabla^2 = U^2 - U^3 + \frac{7}{12}U^4 - \ldots$ and

(ii) $U^2 = \nabla^2 + \nabla^3 + \frac{11}{12}\nabla^4 + \ldots$ (4½,5)

SECTION B

4. (a) If the fourth differences of f(x) can be neglected and $\int_{-1}^{1} f(x) = \frac{2}{3}\left[f(x_1) + f(x_2) + f(x_3)\right]$, find the values of x_1, x_2 and x_3.

(b) Show that the Cote's numbers are symmetric and derive trapezoidal rule using Cote's formula. (4,5½)

5. (a) Show that $\Delta(x\,!) = x(x\,!)$ and hence sum to n terms the series

$$1 + 3\,(2\,!) + 7\,(3\,!) + 13\,(4\,!) + 21\,(5\,!) + \ldots$$

(b) Find the general term and sum to n terms of the following series :

12, 40, 90, 168, 280, 432, ... (4,5½)

6. Solve any **three** of the following difference equations :

(i) $u_{x+1} - au_x = \sin bx$

(ii) $u_{x+3} - 5u_{x+2} + 8u_{x+1} - 4u_x = x.2^x$

(iii) $u_{x+1} - p\,a^{2x} u_x = q\,a^{x^2}$

(iv) $u_{x-1} - 2u_x^2 + 1 = 0$ (3,3,3½)