[This question paper contains 4 printed pages.]

1051 *Your Roll No.* ....

B.Sc. (Hons.) / II **C**

STATISTICS – Paper XII

B–224 : (Probability Theory – II)

(Admissions of 1999 and onwards)

Time : 2 Hours *Maximum Marks : 38*

(Write your Roll No. on the top immediately on receipt of this question paper.)

Attempt **four** *questions in all, selecting at least* **one** *question from each section.*

SECTION I

1. (a) State and prove Chebychev's inequality. Use it to determine how many times a fair coin must be tossed in order that the probability will be at least 0.95 that the ratio of the number of heads to the number of tosses will be between 0.45 and 0.55.

   (b) State and prove De-Moivre-Laplace central limit theorem. Indicate the importance of central limit theorem in statistics. (5,4½)

2. (a) Examine whether the weak law of large numbers is applicable to the sequence $\{X_n\}$ of independent random variables defined as follows :

$$P(X_k = \pm 2^k) = 2^{-(2k+1)},\ P(X_k = 0) = 1 - 2^{-2k}.$$

(b) Define convergence in probability and convergence in distribution. Show that convergence in probability implies convergence in distribution. (4½,5)

SECTION II

3. (a) If X has poisson distribution with parameter λ, λ is also a random variable of the continuous type with the density function :

$$f(\lambda) = a^{\nu} e^{-a\lambda}\ \lambda^{(\nu-1)}/\Gamma\nu\ ;\ \lambda \geq 0,\ \nu > 0,\ a > 0.$$

derive the distribution of X.

(b) If X is a continuous random variable, then how we can obtain its p.d.f. from the characteristic function.

Obtain the p. d. f. of X if its characteristic function is given by $\Phi_X(t) = \exp(-1/2t^2)$. (5,4½)

4. (a) Define r^{th} order statistic. Find the p.d.f. of r th

order statistic. Let $X_1, X_2, \ldots, X_n$ be i.i.d. random variables with distribution function :

$$F(y) = y^{\alpha}, \quad \text{if } 0 < y < 1 \; ; \; \alpha > 0$$

Show that $X_{(i)}/X_{(n)}$, and $X_{(n)}$, $i = 1,2,\ldots, n-1$ are independent.

(b) Let $X_1, X_2, \ldots X_{2n-1}$ be an odd size random sample from a $N(\mu,\sigma^2)$ population. Find the p.d.f. of the sample median and show that it is symmetric about μ. and has the mean μ. (5,4½)

SECTION III

5. (a) If $X_1, X_2, \ldots X_k$ have a multinomial distribution with parameters n and $p_i (i = 1,2,\ldots k)$ with $\Sigma p_i = 1$, obtain the joint probability $P(X_1 = x_1 \cap X_2 = x_2 \cap \ldots \cap X_k = x_k)$. Obtain the corresponding moment generating function. Hence or otherwise show that

$$Cov(X_i, X_j) = -n \, p_i \, p_j ; \; (i \neq j).$$

(b) If $(X, Y) \sim N(0,0,1,1,\rho)$, then prove that

$$P(X > 0, \; Y > 0) = (1/4) + \sin^{-1}\rho / 2\pi \qquad (5,4½)$$

6. (a) Let (X,Y) be a bivariate normal random variable with $E(X) = E(Y) = 0$, $Var(X) = Var(Y) = 1$ and $cov(X,Y) = \rho$. Show that the random variable $Z = Y/X$ has a Cauchy distribution.

(b) Obtain the density function of multivariate normal distribution with mean vector μ and variance covariance matrix Σ. (5,4½)