[This question paper contains 4 printed pages.]

1052   *Your Roll No.* ....

B.Sc. (Hons.) / II   **C**

STATISTICS – PAPER-XIII

B–225 : (Statistical Methods – II)

(Admissions of 1999 and onwards)

Time : 2 Hours   *Maximum Marks : 38*

*(Write your Roll No. on the top immediately
on receipt of this question paper.)*

Attempt **four** *questions in all,
selecting* **two** *questions from each Section.*

SECTION I

1. (a) What is meant by a statistical hypothesis ? What are the two types of errors of decision that arise in testing of a hypothesis ?

   (b) Discuss the test of significance for the difference of standard deviations for large samples.

   (c) Find the minimum sample size n for estimating population proportion P with confidence coefficient $(1 - \alpha)$ and permissible error E in estimate.

                                        (3,3½,3)

                                        P.T.O.

2.   (a) The sex ratio at birth is sometimes given by the ratio of male to female births instead of the proportion of male to total births. If z is the ratio, i.e., $z = p/q$, then show that the standard error of z is approximately $\dfrac{1}{1+z}\sqrt{z/n}$, n being large, so that deviations are small compared with mean.

(b)   (i) Prove that in a random and large sample,

$$\chi^2 = \sum_{i=1}^{k}\left[\frac{(n_i - np_i)^2}{np_i}\right], \text{ follows chi-square}$$

distribution approximately with $(k-1)$ degrees of freedom, where n_i is the observed frequency and np_i is the corresponding expected frequency of the ith class,

$$\left(i = 1, 2..., k\right), \ \sum_{i=1}^{k} n_i = n .$$

(ii) State the conditions for the validity of the above χ^2 test ?   (3½,4+2)

3.   (a) If $X_1, X_2, ..., X_n$ are i.i.d. $U(0,1)$-variates, then show that $-2 \log_e P$ follows chi-square distribution with 2n degrees of freedom where $P = X_1 X_2...X_n$.

(b) Find the distribution of sample correlation coefficient r when the population correlation coefficient $\rho = 0$. Hence deduce that

$$\frac{r}{\sqrt{1-r^2}}\sqrt{n-2}$$

follows Student's t-distribution with (n–2) degrees of freedom.   (9½)

SECTION – II

4.   (a) Explain, stating clearly the assumptions involved, the t-test for testing the significance of the difference between the two sample means.

     (b) If $n_1 = n_2$ in F-distribution, then find the median of F-distribution. Also show that the quartiles Q_1, Q_2 and Q_3 satisfy the condition $Q_1 Q_3 = Q_2$.

     (c) Show that the t-distribution is symmetrical about its mean.   (3½,3,3)

5.   (a) If $X \sim F(1,n)$, Show that $\left(n - \frac{1}{2}\right)\log\left(1 + \frac{x}{n}\right) \sim \chi_1^2$, for large n.

     (b) Let X and Y be two independent normal variates with same normal distribution $N(\mu, \sigma^2)$. Obtain the distribution of

$$Z = \frac{X + Y - 2\mu}{\sqrt{|X - Y|^2}}$$

           (4,5½)

6. (a) Discuss the variate transformation which stabilizes the variance of the distribution of sample variance.

    (b) Define Fisher's Z-transformation and obtain its probability function and moment generating function.

    (c) Let X and Y denote the number of successes and failures respectively in n independent Bernoulli trials with p as probability of success in each trial. Show that

    $$\frac{(X - np)^2}{np} + \frac{\left(Y - n(1 - p)\right)^2}{n(1 - p)} \sim \chi_1^2 \, ,$$

    when n is large. $(3, 4, 2\frac{1}{2})$