[This question paper contains 4 printed pages.]

1054 Your Roll No.

B.Sc. (Hons.) / H

 \mathbf{C}

STATISTICS - Paper XV

B-227: (Sampling Theory and Methods)

(Admissions of 1999 and onwards)

Time: 2 Hours Maximum Marks: 38

(Write your Roll No. on the top immediately on receipt of this question paper.)

Attempt Four questions in all, selecting two questions from each Section.

SECTION A

- (a) Describe the method of determining the sample size in case of simple random sampling so as to meet the desired degree of margin of error and confidence, stating the assumptions made.
 - (b) Define simple random sampling (i) with replacement and (ii) without replacement from a finite

population. Obtain the variances based on the estimators based on the above two methods and compare their efficiencies. (3½,6)

- (a) Establish the result which justifies the statement "Stratification generally leads to greater precision than simple random sampling."
 - (b) If there are two strata and if ϕ is the ratio of the actual n_1/n_2 to the Neyman optimum n_1/n_2 , show that whatever be the values of N_1 , N_2 , S_1 and S_2 , the ratio $V_{\rm opt}(\overline{y}_{\rm st}) / V(\overline{y}_{\rm st})$ is never less than $4\phi(1+\phi)^2$, when the fpc's are negligible.

 $(5,4\frac{1}{2})$

- (a) Discuss the method of collapsed strata. Derive the condition for which this method is as efficient as stratified sampling.
 - (b) Explain the Hansen and Hurwitz technique for removing the bias arising from incomplete samples in mail surveys and obtain the variance of the estimator of the population mean. (3½,6)

SECTION B

4. (a) Derive, to the first approximation, the expressions for the bias and variance of the ratio estimator.

(b) Stating clearly the underlying assumptions, show that for a super population model, the regression estimator is BLUE. Also derive the expression for the minimum variance. (4.5½)

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- 5. (a) Define systematic and circular systematic sampling. Prove that systematic sampling is more precise than SRS if the variation within the systematic sample is larger than the population variation as a whole.
 - (b) Obtain the variance of the regression estimate in double sampling which is minimum for a given cost and find its relative efficiency with respect to the unbiased estimate based on SRS. (4½,5)
- 6. (a) Establish the results which justify the following statements:
 - (i) Efficiency of cluster sampling increases as mean square within clusters increases.
 - (ii) If clusters are formed of random samples of elements of population, they on the average be as efficient as the individual elements themselves.

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(b) Prove that the two stage sampling is more efficient than one stage sampling if $\rho < 0$, where ρ is the intraclass correlation coefficient between the elements of the first stage units (equal first stage units). (5½,4)