[This question paper contains 5 printed pages.]

1791 Your Roll No. ....

B.Sc. (Hons.) III Sem./II Yr. **A**

Paper 304 : STATISTICS

(Admissions of 2001 and onwards)

Time : 3 Hours *Maximum Marks : 75*

(Write your Roll No. on the top immediately on receipt of this question paper.)

Attempt **all** *questions.*
All *questions carry equal marks.*

Use of scientific calculator and statistical table is allowed in the examination.

1. The median and mode of the following wage distribution are known to be Rs. 3,350 and Rs. 3,400 respectively. Find the values of f_3, f_4 and f_5 :

Wage (in Rs.)	0-1000	1000-2000	2000-3000	3000-4000	4000-5000	5000-6000	6000-7000	Total
No. of Employees	4	16	f_3	f_4	f_5	6	4	230

2. For a distribution, Bowley's coefficient of skewness is –0.56, Q_1 = 16.4, Median = 24.2. Find the coefficient of Quartile deviation.

P.T.O.

3. The first three moments of distribution about value 2 are 1, 16 and −40 respectively. Find mean, variance and third moment (μ_3). Also obtain first three moments about zero.

4. The first of two samples has 100 items with mean 15 and standard deviation 3. If the whole group has 250 items with mean 15.6 and variance 13.44, find the variance of second group.

5. Ten competitors in a musical test were ranked by three judges A, B, C in the following order :

Ranks by A	1	6	5	10	3	2	4	9	7	8
Ranks by B	3	5	8	4	7	10	2	1	6	9
Ranks by C	6	4	9	8	1	2	3	10	5	7

Using rank correlation method, discuss which pair of judges has the nearest approach to common linkings in music.

6. Show that

$$1 - R_{1.23}^2 = (1 - r_{12}^2)(1 - r_{13.2}^2)$$

(i) Deduce that

$$1 - R_{1.23}^2 = \frac{(1-p)(1+2p)}{(1+p)}, \text{ provided}$$

all coefficients of zero order are equal top.

(ii) If $R_{1.23} = 0$, show that

$$r_{12} = r_{13} = 0$$

7. If the joint density of X_1, X_2 and X_3 is given by

$$f(x_1, x_2, x_3) = \begin{cases} (x_1 + x_2)e^{-x_3}; & 0 < x_1 < 1, \\ & 0 < x_2 < 1, x_3 > 0 \\ 0 ; & \text{otherwise} \end{cases}$$

find the regression equation of X_2 on X_1 and X_3.

8. Consider the random variable X with p.d.f.

$$f(x) = b \exp[-b(x-a)]; \; a < x < \infty, \; b > 0$$

find the measure of skewness based on Quartiles. Also find the measure of Kurtosis (β_2).

9. Suppose the service life, in hours, of a semi-conductor is a random variable having a weibull distribution :

$$f(x) = \begin{cases} k x^{\beta-1} . e^{-\alpha x^{\beta}}; & x > 0 \\ 0 ; & \text{otherwise} \end{cases}$$

with $\alpha = 0.025$ and $\beta = 0.500$.

(a) How long can such a semiconductor be expected to last ?

(b) What is the probability that such a semiconductor will still be in operating condition after 4,000 hours.

10. If $X_1, X_2, ---- X_n$ constitute a random sample from a uniform population with p.d.f.

$$f(x, \beta) = \frac{1}{\beta}, \; 0 < x < \beta$$

(a) Show that $\frac{n+1}{n}Y_n$ and $2\overline{X}$ are both unbiased estimators for β.

(b) Compare the efficiency of the two estimators :

$\frac{n+1}{n}Y_n$ and $2\overline{X}$

Interpret your result.

11. If $X_1, X_2, ----, X_n$ constitute a random sample from the population :

$$f(x) = \begin{cases} e^{-(x-\delta)}; & x > \delta \\ 0 \;; & \text{elsewhere} \end{cases}$$

Show that Y_1 (the first order statistic) is a consistent estimator of parameter δ.

12. It is assumed that concentration of a pollutant, in parts per million, has a log-normal distribution with parameters $\mu = 3.2$ and $\sigma = 1$. What is the probability that the concentration exceeds 9 parts per million ?

13. Use the data in the following table to test at 0.01 level of significance whether a person's ability in mathematics is independent of his or her interest in statistics.

		Ability in Mathematics		
		Low	Average	High
Interest in Statistics	Low	63	42	15
	Average	58	61	31
	High	14	47	29

(Given $\chi^2_{0.01}(4) = 13.277$)

14. Twelve pieces of material I were tested by exposing each piece to a machine measuring wear. Ten pieces of material II were similarly tested. In each case, the depth of were was observed. The samples of material I gave an average (coded) wear of 85 units with sample standard deviation of 4, while the samples of material II gave an average of 81 and a sample standard deviation of 5. Can we conclude at 0.05 level of significance that abrasive wear of material I exceeds that of material II by more than 2 units ?

(Given $t_{0.10}(20) = 1.325$)

15. If $x \geq 1$ is the critical region for testing $H_0 : \theta = 2$ against the alternative $H_1 : \theta = 1$ on the basis of the single observation from the population,

$$f(x, \theta) = \theta \exp(-\theta x), \ 0 \leq x < \infty$$

Obtain the values of Type I and Type II errors.