[This question paper contains 4 printed pages.]

Sr. No. of Question Paper : 8787 **C** **Roll No............**

Unique Paper Code : 237301

Name of the Paper : STHT-302 : Probability and Statistical Methods – III

Name of the Course : B.Sc. (Hons.) Statistics, Part II

Semester : III

Duration : 3 Hours

Maximum Marks : 75

Instructions for Candidates

1. Write your Roll No. on the top immediately on receipt of this question paper.
2. Attempt **five** questions selecting two from section I and three from section II.

SECTION I

1. (a) If U = aX + bY and V = cX + dY, where X and Y are measured from their respective means and r is the correlation coefficient between X and Y, and if U and V are uncorrelated, show that

$$\sigma_U \sigma_V = (ad - bc)\sigma_X \sigma_Y \sqrt{1-r^2}\ .$$

(b) Define Spearman's rank correlation coefficient. Obtain the value of rank correlation coefficient when each of the deviations is maximum. (7,8)

2. (a) The regression lines of Y on X and of X on Y are Y = aX + b and X = cY + d. Show that

(i) means are $\bar{X}$ = (bc+d)/(1–ac) and $\bar{Y}$ = (ad+b)/(1–ac),

(ii) correlation coefficient r(X,Y) = $\sqrt{ac}$,

(iii) The ratio of the standard deviations of X and y is $\sqrt{c/a}$.

(b) Show that, in the usual notations

$$1 - R_{1.23}^2 = \left(1 - r_{12}^2\right)\left(1 - r_{13.2}^2\right)$$

Deduce that

(i) $R_{1.23} \geq r_{12}$

(ii) $R_{1.23}^2 = r_{12}^2 + r_{13}^2$, if $r_{23} = 0$

(iii) $1 - R_{1.23}^2 = \dfrac{(1-\rho)(1+2\rho)}{(1+\rho)}$, provided all coefficients of zero order are equal to ρ. (7,8)

3. (a) If X and Y are two independent random variables such that

$f(x) = e^{-x}$, $x \geq 0$, and $g(y) = 3e^{-3y}$, $y \geq 0$.

Find the probability distribution of $Z = X/Y$.

(b) Explain the concept of partial correlation. In a trivariate distribution, show that

(i) $\sigma_{1.23}^2 = \sigma_1^2 \omega / \omega_{11}$,

(ii) $r_{12.3} = \dfrac{r_{12} - r_{13} r_{23}}{\sqrt{1 - r_{13}^2}\sqrt{1 - r_{23}^2}}$ (7,8)

SECTION II

4. (a) Define convergence in probability and convergence with probability one. Show that convergence with probability one implies convergence in probability.

(b) Let $\{X_n\}$ be a sequence of independent identically distributed random variables and $S_n = X_1 + X_2 + \ldots\ldots + X_n$, show that

$$\lim_{n\to\infty} P\left[a \le \frac{S_n - E(S_n)}{\sqrt{V(S_n)}} \le b\right] = \Phi(b) - \Phi(b),\ where\ \Phi(.)$$

is the distribution function of a standard normal variate. (7,8)

5. (a) Let g(X) be a non negative function of a random variable X, then for every $k > 0$, show that

$$P[g(X) \ge k] \le \frac{E(g(X))}{k}$$

Hence obtain chebyshev's inequality from it.

(b) The trinomial distribution of two random variables X and Y is given by :

$$f(x, y) = \frac{n!}{x!y!(n-x-y)!} p^x q^y (1-p-q)^{n-x-y},\ x,y = 0, 1, 2, \ldots, n;\ x + y \le n,$$

$0 \le p, q, p + q \le 1.$

Find the marginal distribution of X and the conditional distribution of Y given X = x. Also obtain correlation between X and Y. (7,8)

6. (a) Let X and Y be standard normal variates with coefficient of correlation ρ. Show that

$$\mu_{r,s} = (s+r-1)\rho\, \mu_{r-1,s-1} + (r-1)(s-1)(1-\rho^2)\mu_{r-2,s-2}.$$

Hence deduce that $\mu_{31} = 3\rho$ and $\mu_{22} = 1 + 2\rho^2$.

(b) If the variables $X_1, X_2, \ldots, X_n$ are uniformly bounded then prove that the condition $\lim_{n\to\infty} \frac{V(X_1 + X_2 + \ldots + X_n)}{n^2} = 0$ is necessary as well as sufficient for weak law of large numbers to hold. (7,8)

7. (a) If $X \sim N_p(\mu, \Sigma)$, find the characteristic function of X.

(b) State Inversion theorem for characteristic function. Let X be a discrete random variable with its characteristic function given by

$$\phi_X(t) = e^{\lambda(e^{it}-1)}$$

Obtain the probability function of X.

(c) Write a note on sin inverse and logarithmic transformations. (5,5,5)