[This question paper contains 4 printed pages.]

Sr. No. of Question Paper : 8786 **C** **Roll No............**

Unique Paper Code : 237352

Name of the Paper : STHT-301 : Real Analysis

Name of the Course : B.Sc. (Hons.) Statistics, Part II

Semester : III

Duration : 3 Hours

Maximum Marks : 75

Instructions for Candidates

1. Write your Roll No. on the top immediately on receipt of this question paper.
2. Attempt **six** questions in all.
3. Question No. **1** is compulsory.

1. (a) Write down the supremum and infimum of the following, if these exist

$$T = \left\{ m + \frac{1}{n} : m, n \in N \right\}.$$ (2)

(b) Let S be a non empty subset of R such that sup. S = Inf. S. What can be said about the set ? (1)

(c) Give an example of a set which is not a neighbourhood of any of its points. (1)

(d) Give an example of an interval which is an open set. (1)

(e) Is the union of an arbitrary family of closed sets a closed set ? (2)

(f) Show that the series 2 – 2 + 2 – 2 + ... oscillates. (2)

(g) Assuming that $n^{1/n} \to 1$ as $n \to \infty$, show by applying Cauchy's n^{th} root test that the series $\sum_{n=1}^{\infty} \left(n^{1/n} - 1\right)^n$ converges. (2)

(h) Is the sequence $<a_n>$, where $a_n = 1 + \frac{(-1)^n}{n}$ $\forall n \in N$ convergent ? If yes, justify your answer. (2)

(i) Examine for continuity the function

$$f(x) = \begin{cases} \frac{|x|}{x} & if\ x \neq 0 \\ 1 & if\ x = 0 \end{cases}$$

at the origin. (2)

2. (a) Define upper bound and supremum of a set of real numbers. State the order completeness property of real numbers and apply it to prove that any non empty set of real numbers which is bounded below has an infimum. (6)

(b) Define an open set. Show that the union of an arbitrary family of open sets is open but the intersection of an arbitrary family of open sets may fail to be an open set. (6)

3. (a) Define

(i) Neighbourhood of a point

(ii) Closed set

(iii) Limit point of a set

Show that a finite set has no limit point. (6)

(b) Let S and T be arbitrary subsets of R, then prove that

(i) $\varphi' = \varphi$

(ii) $S \subset T \Rightarrow S' \subset T'$

(iii) $(S \cup T)' = S' \cup T'$ (6)

4. (a) State Monotone Convergence Theorem and hence prove that the sequence $<a_n>$ defined by the relation, $a_1 = 1$, $a_n = 1 + \frac{1}{1!} + \frac{1}{2!} + \ldots + \frac{1}{(n-1)!}$ $(n \geq 2)$ converges. (6)

(b) State and prove Cauchy's First Theorem on limits for sequences. (6)

5. (a) State and prove D'Alemberts Ratio Test for positive term series. (6)

(b) Test for convergence the series

$$\frac{1}{2}x + \frac{1.3}{2.4}x^2 + \frac{1.3.5}{2.4.6}x^3 + \ldots + \frac{1.3\ldots(2n-1)}{2.4\ldots(2n)}x^n + \ldots$$

for all positive values of x. (6)

6. (a) Define absolutely convergent series. Show that every absolutely convergent series is convergent. Is the converse true ? Justify you answer. (4)

(b) Examine the continuity of the function at x = 0, which is defined as follows :

$$f(x) = \begin{cases} \dfrac{e^{1/x} - e^{-1/x}}{e^{1/x} + e^{-1/x}} & \text{when } x \neq 0 \\ 0 & \text{when } x = 0 \end{cases}$$ (4)

(c) Let f be defined on R by setting $f(x) = |x - 2| + |x + 2|$ for all $x \in R$. Show that f is derivable at every point except at $x = 2$ and $x = -2$. (4)

7. (a) State and prove Cauchy's Mean Value Theorem. (6)

(b) Solve any **three** of the following :

(i) $\lim_{x \to \frac{\pi}{2}} \dfrac{\cos x}{\frac{\pi}{2} - x}$

(ii) $\lim_{x \to 0} \dfrac{\tan x - x}{x^2 \tan x}$

(iii) $\lim_{x \to \infty} \left(\dfrac{x^n}{e^x}\right)$, n being positive integer

(iv) $\lim_{x \to 0} \left(\dfrac{1}{x} - \dfrac{1}{\sin x}\right)$ (6)