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Sr. No. of Question Paper : 8789

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Roll No.....

Unique Paper Code : 237303

Name of the Paper : STHT-304 : Survey Sampling

Name of the Course : B.Sc. (Hons.) Statistics, Part II

Semester : III

Duration : 3 Hours

Maximum Marks : 75

**Instructions for Candidates**

1. Write your Roll No. on the top immediately on receipt of this question paper.
2. Attempt **five** questions in all, selecting **three** from Section A and **two** from Section B.

**SECTION A**

1. (a) What is the need of sample survey ? Compare it with census explaining its advantages and disadvantages.  
(b) Define simple random sampling (i) with replacement and (ii) without replacement from a finite population. Obtain the variances based on the estimators based on the above two methods and compare their efficiencies.  
(c) From a simple random sample of  $n$  units drawn without replacement from a population of  $N$  units, a simple random sub-sample of  $m$  units is drawn and added to the original sample. Show that mean based on  $(n + m)$  units is an unbiased estimator of the population mean. Obtain the variance of this estimator. Compare the efficiency of this estimator with the one based on original  $n$  units. (3,6,6)

P.T.O.

2. (a) What are the practical difficulties in adopting Neyman method of allocation? Determine the size of preliminary sample for which the modified Neyman allocation will always be more efficient than proportional allocation.
- (b) Discuss the method of collapsed strata. Derive the condition for which this method is as efficient as stratified sampling.
- (c) Show that if a random sample of size  $n$  is drawn without replacement from a finite population of size  $N$  with mean  $\mu$  and mean square  $S^2$ , the covariance between any two elements of the sample is  $-S^2/N$ . (7,5,3)
3. (a) Explain systematic sampling and state the situations where it can be used. Show that a systematic sample has the same precision as the corresponding stratified sample with one unit per stratum if  $\rho_{wst} = 0$ , where  $\rho_{wst}$  is the coefficient of correlation between the deviations from the stratum means of the pairs of items that are in the same systematic sample.
- (b) Describe the method of allocating a sample to different strata. Also obtain the variance of the estimate of the population mean under these allocations. Hence establish :

$$V(\bar{y}_{st})_{Ney} \leq V(\bar{y}_{st})_{prop} \leq V(\bar{y}_n). \quad (9,6)$$

4. (a) Explain the following terms with reference to sample survey :  
Sampling unit, Frame, Optimisation and Validity.
- (b) The right triangular distribution  $f(y) = 2(1-y)$ ;  $0 < y < 1$  is divided into two strata at the point  $a$ . Show that :

$$p_1 = a(2-a), \quad p_2 = (1-a)^2;$$

$$S_1^2 = \frac{a^2(6 - 6a + a^2)}{18(2 - a)^2}, \quad S_2^2 = \frac{(1 - a)^2}{18}$$

where  $p_i$  is proportion and  $S_i^2$  is the variance of the  $i$ th stratum.

- (c) What is meant by non-response in sample surveys? Explain the Hansen and Hurwitz technique for removing the bias arising from non-response in mail surveys. Obtain the variance of the estimator of the population mean and minimize it for the fixed cost. (4,4,7)

### SECTION B

5. (a) Stating clearly the underlying assumptions, show that for a super population model, the ratio estimator is BLUE. Also derive the expression for the minimum variance.
- (b) Show that ratio estimator is biased. Obtain an upper bound to its bias.
- (c) Derive to the first approximation, the expressions for the bias and variance of the linear regression estimator. (6,2,7)
6. (a) Establish the result which justify the statement "Efficiency of cluster sampling as compared to simple random sampling without replacement increases with the increase of mean squares within clusters." Also obtain an estimate of this relative efficiency.
- (b) Prove that the two stage sampling is more efficient than one stage sampling if  $\rho < 0$ , where  $\rho$  is the intraclass correlation coefficient between the elements of the first stage units (equal first stage units). (6,9)

7. (a) Discuss the problem of determining the optimum size of cluster which will provide the maximum precision for the fixed cost.
- (b) Compare linear regression estimator with ratio estimator and simple random sampling without replacement. (Assume the formulae for the variance of the estimators). (10,5)