[This question paper contains 4 printed pages.]

Sr. No. of Question Paper : 6701 **D** **Your Roll No...............**

Unique Paper Code : 237352

Name of the Course : **B.Sc. (Hons.) Statistics**

Name of the Paper : STHT–301 : Real Analysis

Semester : III

Duration : 3 Hours Maximum Marks : 75

Instructions for Candidates

1. Write your Roll No. on the top immediately on receipt of this question paper.
2. Q. No. **1** is compulsory.
3. Attempt **six** questions in all.

1. (a) Write down the supremum and infimum of the following :

$$S = \left\{1 - \frac{(-1)^n}{n} : n \in N\right\}$$ (1)

(b) Give an example of a set which is

(i) infinite and bounded

(ii) neither bounded above nor bounded below (2)

(c) Write down the set of all limit points of

(i) set Z of integers

(ii) set of rational numbers (2)

(d) (i) Give an example of a bounded sequence which is not convergent.

(ii) If $\lim_{n\to\infty} a_n = 1$ and if $< S_n >$ be the sequence defined as

$S_n = (a_1 + a_2 + ... + a_n)/n$, then, find $\lim_{n\to\infty} S_n$. (1,1)

P.T.O.

(e) Show that the series $1+\frac{2}{3}+\left(\frac{2}{3}\right)^2+...+\left(\frac{2}{3}\right)^{n-1}$ converges. (1)

(f) What is an alternating series ? Give an example. (1)

(g) Examine the continuity at $x = 0$ for the function defined as

$$f(x) = \begin{cases} \frac{\sin 2x}{\sin x}, & x \neq 0 \\ 1 \quad , & x = 0 \end{cases}$$ (2)

(h) Give the geometrical interpretation of Rolle's Theorem. (2)

(i) For what values of '**a**' does $\frac{\sin 2x + a \sin x}{x^3}$ tend to a finite limit as $x \to 0$. Also find this limit. (2)

2. (a) Let S be a non empty set of real numbers bounded below. Then prove that a real number t is the infimum of S iff the following conditions hold

 (i) $x \geq t, \quad \forall\, x \in S$

 (ii) for each positive real number ε, there is a real number $x \in S$ such that $x < t + \varepsilon$.

 (b) If M and N are neighbourhoods of a point p, then prove that $M \cap N$ is also a neighbourhood of p. (6,6)

3. (a) Define the following :

 (i) an open set.

 (ii) a monotonic sequence

 (iii) a bounded set

 (b) Show that the intersection H of an arbitrary family ξ of closed sets is a closed set.

(c) Show that a set is closed iff it contains all its limit points. (3,3,6)

4. (a) Prove that $\lim_{n\to\infty}\left(1+\frac{1}{n}\right)^n$ exists and lies between 2 and 3.

(b) Show that every Cauchy sequence is bounded. (6,6)

5. (a) Show that the series $\sum_{n=1}^{\infty} 1/n^p$ converges if $p > 1$ and diverges if $p \le 1$.

(b) Test for convergence of the series

$$\sum_{n=1}^{\infty} \frac{\sqrt{n}}{\sqrt{n^2+1}} x^n$$ for positive values of x. (6,6)

6. (a) Define conditionally convergent series. Show that the series

$1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + \ldots$ is conditionally convergent.

(b) Let f be a function defined on [0,1] as

$f(x) = \frac{1}{2^n}$, when $\frac{1}{2^{n+1}} \le x \le \frac{1}{2^n}$, $n = 0,1,2,3,\ldots$

$f(0) = 0$

Show that f is continuous except at the points $x = \frac{1}{2}, \frac{1}{2^2}, \ldots, \frac{1}{2^n}, \ldots$. Describe the nature of discontinuity at each of these points. (6,6)

7. (a) Prove that for any quadratic function $px^2 + qx + r$, the value of θ in Lagrange's Mean Value Theorem is always ½, whatever p, q, r, a and h may be.

(b) Obtain Maclaurin's series expansion of sin x.

(c) Evaluate any **two** of the following :

(i) $\lim_{x \to 0}(1 + x)^{1/x}$

(ii) $\lim_{x \to 0}(x^k \log x)$ for each fixed $k \in R^+$

(iii) $\lim_{x \to 0}(\tan x)^{\sin 2x}$ (3,3,2,2,2)