[This question paper contains 2 printed pages.]

Sr. No. of Question Paper : 6702 **D** **Your Roll No...............**

Unique Paper Code : 237301

Name of the Course : **B.Sc. (Hons.) Statistics**

Name of the Paper : STHT–302 : Probability and Statistical Methods – III

Semester : III

Duration : 3 Hours Maximum Marks : 75

Instructions for Candidates

1. Write your Roll No. on the top immediately on receipt of this question paper.
2. Attempt **five** questions selecting **two** from **Section I** and **three** from **Section II.**

SECTION I

1. (a) If X and Y are uncorrelated random variables with zero means and variances σ_1^2 and σ_2^2 respectively, then show that $U = X\cos\alpha + Y\sin\alpha$ and $V = X\sin\alpha - Y\cos\alpha$, have a correlation coefficient given by

$$\frac{\sigma_1^2 - \sigma_2^2}{\left[\left(\sigma_1^2 - \sigma_2^2\right)^2 + 4\sigma_1^2\sigma_2^2 \operatorname{cosec}^2 2\alpha\right]^{1/2}}.$$

(b) If d_i is the difference in the ranks of the i^{th} individual for $i = 1,2,$ ------, n in two different characteristics, then show that the maximum value of $\sum_{i=1}^{n} d_i^2$ is $\frac{1}{3}\left(n^3 - n\right)$. Hence or otherwise, show that rank correlation coefficient lies between −1 and +1. (8,7)

2. (a) Derive the equation of the lines of regression and show that the coefficient of correlation is the geometric mean of the regression coefficients.

(b) In a trivariate distribution, in the usual notations, show that $\sigma_{1.23}^2 = \sigma_1^2 \frac{\omega}{\omega_{11}}$.

Hence or otherwise show that $\sigma_1^2 \geq \sigma_{1.2}^2 \geq \sigma_{1.23}^2$. (8,7)

3. (a) Show that if $X_3 = aX_1 + bX_2$, then the three partial correlation coefficients are numerically equal to unity, $r_{13.2}$ having the sign of a, $r_{23.1}$ having the sign of b and $r_{12.3}$, having the sign opposite to that of a/b.

(b) Let $X \sim \beta_1(\mu, v)$ and $Y \sim \gamma(\lambda, \mu + v)$ be two independent random variables, $(\mu, v, \lambda > 0)$. Find the p.d.f. of XY and identify the resulting distribution. (8,7)

SECTION II

4. (a) State and prove De–Moivre-Laplace central limit theorem.

(b) If X is a non-negative random variable and if E(X) exists, then for any $a > 0$, prove that $P[X \geq a] \leq \frac{E(X)}{a}$.

Use this result to prove that in 1000 tosses of a coin the probability that the number of tails lies between 450 and 550 is at least 0.9. (7,8)

5. (a) Define various modes of convergence of a sequence $\{X_n\}$ of random variables to a random variable X. Show that convergence with probability one implies convergence in probability.

(b) If the random variables $X_1, X_2, \ldots, X_k$ have a multinomial distribution then obtain the marginal distribution of X_i. Also find $E(X_i)$, $cov(X_i, X_j)$ and the correlation coefficient between X_i and X_j. $(\forall i, j = 1,2,\ldots,k)$. (7,8)

6. (a) Let X be a continuous random variable with characteristic function $\varnothing_x(t) = e^{\frac{-t^2}{2}}$. Obtain the probability density function of X.

(b) Show, by means of an example that the normality of conditional p.d.f.'s does not imply that the bivariate density is normal.

(c) If $(X, Y) \sim BVN(\mu_1, \mu_2, \sigma_1^2, \sigma_2^2, \rho)$, then obtain the correlation coefficient between e^x and e^y. (5,5,5)

7. (a) If $\underset{\sim}{X} \sim N_p\left(\underset{\sim}{\mu}, \Sigma\right)$ then find the moment generating function of $\underset{\sim}{X}$.

(b) Let $\{X_k\}$, k=1,2,3,.... be mutually independent and identically distributed random variables with mean μ and finite variance σ^2. If $S_n = X_1 + X_2 + \ldots + X_n$, examine whether the weak law of large numbers hold for the sequence $\{S_n\}$.

(c) Write short notes on Fisher's-Z and sin inverse transformations. (5,5,5)

(600)