This question paper contains 2 printed pages. **Your Roll No. ....**

Sl. No. of Ques. Paper	: **1472**	**F-7**
Unique Paper Code	: **2371302**	
Name of Paper	: **Survey Sampling**	
Name of Course	: **B.Sc. (H) Statistics (Erstwhile FYUP)**	
Semester	: **III**	
Duration	: **3 hours**	
Maximum Marks	: **75**	

(Write your Roll No. on the top immediately on receipt of this question paper.)

*Attempt **five** questions in all, selecting **three** from Section I and **two** from Section II.*

SECTION I

1. (a) What are the main steps involved in a sample survey?

   (b) Define simple random sampling (i) with replacement and (ii) without replacement from a finite population. Obtain the variances of estimators based on the above two methods and compare their efficiencies.

   (c) For srswor, prove that:

$$\text{cov}(x_i, \bar{y}_n) = \frac{N-n}{Nn} \cdot \frac{1}{N-1} \sum_{i=1}^{N} (X_i - \bar{X}_N)(Y_i - \bar{Y}_N) = \frac{N-n}{n(N-1)} \text{cov}(X, Y)$$

3,6,6

2. (a) What is the guiding principle for construction of strata? Obtain the optimal points of stratification with Neyman allocation and proportional allocation.

   (b) Explain systematic sampling with example. Show that a systematic sample has the same precision as the corresponding stratified random sample with one unit per stratum if $\rho_{wst} = 0$, the notation has its usual meaning.

9,6

3. (a) Justify the following statements:

   (i) The smaller the size of stratum, the smaller should be the size of sample to be selected therefrom.

   (ii) The smaller the variability within a stratum, the smaller should be the size of sample selected from the stratum.

   (iii) The cheaper the cost per unit in a stratum, the larger should be the size of sample selected from that stratum.

   Hence obtain minimum size required for estimating population mean with fixed variance under optimum allocation.

(b) With two strata, a surveyor would like to have $n_1 = n_2$ for administrative convenience instead of using the values given by Neyman's allocation. If $V(\bar{y}_{st})$ and $V(\bar{y}_{st})_{opt}$ denote the variances given for $n_1 = n_2$ and Neyman's allocation respectively, show that the fractional increase in the variance is

$$\frac{V(\bar{y}_{st}) - V(\bar{y}_{st})_{opt}}{V(\bar{y}_{st})_{opt}} = \left(\frac{r-1}{r+1}\right)^2$$

where $r = n_{1(opt)}/n_{2(opt)}$ and f.p.c. are ignored. 8,7

4. (a) What is meant by non-response in sample surveys? What is the effect of non-response on the estimates? Prove the statement "the estimate obtained from incomplete samples may be reasonable if response and non-response classes are alike."

(b) Prove that in systematic sampling, positive intra class correlation coefficient between units of same systematic sample inflates the variance of the systematic sample mean.

(c) Explain the following terms with reference to sample survey:
Sampling unit and frame, Optimisation and Validity. 6,6,3

SECTION II

5. (a) Stating clearly the underlying assumptions, show that the ratio estimator under a super population model is BLUE. Also derive the expression for the minimum variance.

(b) Derive to the first approximation, the expressions for the bias and variance of the linear regression estimator. 9,6

6. (a) Prove that the mean of cluster means $\bar{\bar{y}}$ is an unbiased estimator of population mean with variance given as

$$V\left(\bar{\bar{y}}\right) = \frac{N-n}{N-1} \cdot \frac{\sigma^2}{nM}\left[1 + (M-1)\rho\right]$$

(b) For two-stage sampling, if the total cost of survey is proportional to the size of the sample, discuss the problem of determining the optimal values of n and m to estimate the population mean with maximum precision for given cost or has desired precision for minimum cost. 9,6

7 (a) If y and x are unbiased estimators of the population totals of Y and X respectively, show that the variance of ratio estimate $\frac{y}{x}$ can be approximated by $c_y^2 - c_x^2$, where c_x and c_y are coefficient of variation of x and y respectively. (The correlation coefficient between $\frac{y}{x}$ and x is assumed to be negligible.)

(b) Compare two-stage sampling with cluster sampling and simple random sampling without replacement. 6,9