[This question paper contains 4 printed pages.]

Sr. No. of Question Paper : 1147 **G** **Your Roll No...............**

Unique Paper Code : 237301

Name of the Paper : Probability and Statistical Methods III (STHT-302)

Name of the Course : **B.Sc. (Hons.) Statistics**

Semester : III

Duration : 3 Hours Maximum Marks : 75

Instructions for Candidates

1. Write your Roll No. on the top immediately on receipt of this question paper.
2. Attempt **5** questions in all selecting any **2** questions from **Section A** and any **3** questions from **Section B.**

SECTION A

1. (a) Define Karl Pearson's coefficient of correlation. Show that coefficient of correlation is independent of change of origin and scale. Also prove that for two independent variables r = 0. Show by an example that the converse is not true.

   (b) If U = aX + bY and V = bX – aY, then show that U and V are uncorrelated if

$$\frac{ab}{\left(a^2-b^2\right)}=\frac{\rho\sigma_x\sigma_y}{\sigma_x^2-\sigma_y^2}$$

where ρ is the coefficient of correlation between X and Y. Show further that

$$\sigma_u^2+\sigma_v^2=\left(a^2+b^2\right)\left(\sigma_x^2+\sigma_y^2\right)$$ (7,8)

2. (a) Define Spearman's rank correlation coefficient. Obtain the value of rank correlation coefficient when each of the deviation is maximum. Also briefly discuss the case for tied ranks.

(b) The equations of the two regression lines are :

$X + 2Y - 5 = 0; \;\; 2X + 3Y - 8 = 0$

Obtain

(i) The value of correlation coefficient

(ii) Mean values of X and Y

(iii) Var(Y) if Var(X) = 12 (8,7)

3. (a) Explain the concept of partial correlation coefficient. Show that in the usual notations

$$r_{12.3} = \frac{r_{12} - r_{13}r_{23}}{\sqrt{\left(1-r_{13}^2\right)\left(1-r_{23}^2\right)}}$$

(b) Let X and Y be independent random variables with common p.d.f.

$$f(x) = \begin{cases} e^{-x}, & x \geq 0 \\ 0 \;, & x < 0 \end{cases}.$$

Find the p.d.f. of X–Y. (8,7)

SECTION B

4. (a) State and prove Chebyshev's inequality. For geometric distribution

$p(x) = 2^{-x}, \; x = 1,2,3\ldots,$

Prove that Chebyshev's inequality gives $P\left[\,|x-2| \leq 2\,\right] > \frac{1}{2}$.

(b) State and prove Lindberg-Levy Central Limit Theorem. (8,7)

5. (a) Define convergence in probability and convergence with probability one. Show that convergence with probability one implies convergence in probability.

(b) If the variables $X_1, X_2 \ldots\ldots\ldots X_n$ are uniformly bounded then prove that condition

$\lim_{n\to\infty} \frac{V(X_1 + X_2 + \cdots + X_n)}{n^2} = 0$ is necessary as well as sufficient for weak law of large numbers to hold. (7,8)

6. (a) State inversion theorem for characteristic function. Find the probability mass function of the random variable X for which the characteristic function is

$$\varnothing_X(t) = e^{\lambda(e^{it}-1)}$$

(b) For two independent standard normal variates X and Y, obtain the m.g.f. of X.Y.

(c) Write short notes on Fisher's Z transformation and Square root transformation. (5,5,5)

7. (a) Find the characteristic function of $\underset{\sim}{X} \sim N_p(\mu, \Sigma)$.

(b) If $X_1, X_2, X_3, \ldots\ldots, X_K$ are K independent Poisson variates with parameters $\lambda_1, \lambda_2, \lambda_3, \ldots\ldots, \lambda_K$ respectively, prove that the conditional distribution $P(X_1 \cap X_2 \cap \ldots\ldots \cap X_K | X\}$ where $X = X_1 + X_2 + \ldots + X_K$ is fixed, is multinomial.