[This question paper contains 4 printed pages.]

Sr. No. of Question Paper : 1146 **G** **Your Roll No...............**

Unique Paper Code : 237352

Name of the Paper : Real Analysis (STHT-301)

Name of the Course : **B.Sc. (Hons.) Statistics**

Semester : III

Duration : 3 Hours Maximum Marks : 75

Instructions for Candidates

1. Write your Roll No. on the top immediately on receipt of this question paper.
2. Q.No. **1** is compulsory.
3. Attempt **six** questions in all.

1. (a) Write down the Supremum and Infimum of the following :

$$S = \left\{1 - \frac{1}{n},\ n \in N\right\}$$

(b) (i) Give an example of an infinite closed set which is not an interval.

(ii) Give an example of a set which is neither an interval nor an open set.

(c) Show that arbitrary intersection of open sets need not be open.

(d) Examine the convergence of the series $\sum \cos\frac{1}{n}$.

(e) The interval [1 2 [is not a closed set. Justify.

(f) Examine the continuity of the function $f(x) = [x], x \in R$ at $x = 2$.

(g) Find the value of 'c' of Lagrange Mean Value Theorem for the function

$f(x) = 1/x$ on [1 4]. (2,2,2,2,2,3,2)

2. (a) Define infimum of a non empty bounded set S of real numbers. Prove that a real number t is the infimum of S iff

(i) $x \geq t \quad \forall x \in S$

(ii) For each $\in > 0$, there is a real number $x \in S$ such that $x < t + \in$.

(b) Define limit point of a set. Prove that a set is closed if it contains all its limit points.

(c) Write 'True' or 'False' :

(i) The set of integers has no limit point

(ii) The interval [1,2] is open set. (5,5,2)

3. (a) State and prove Cauchy's General Principle of convergence of a sequence.

(b) State Cauchy's first theorem on limits. Use it to show that

$$\lim_{n\to\infty} \frac{1 + 2^{1/2} + 3^{1/3} + ... + n^{1/n}}{n} = 1$$ (6,6)

4. (a) State Monotone Convergence Theorem. Use it to show that the sequence $<a_n>$, defined by the relation as

$$a_1 = 1,\ a_n = 1+\frac{1}{1!}+\frac{1}{2!}+\ldots+\frac{1}{(n-1)!},\quad n \geq 2 \text{ converges.}$$

(b) Define

(i) a convergent series

(ii) a divergent series

Give one example of each of the above series. Show that the series $1 + r + r^2 + \ldots + \ldots$ converges if $0 < r < 1$. (6,6)

5. (a) Let $\sum u_n$ be a positive term series such that $\lim_{n\to\infty} \frac{u_n}{u_{n+1}} = l$. Prove that the series $\sum u_n$ converges if $l > 1$. What happens if $l = 1$.

(b) Examine the convergence of the following series :

(i) $\sum_{n=1}^{\infty} 2^{-n-(-1)^n}$

(ii) $\frac{x}{2\sqrt{3}}+\frac{x^2}{3\sqrt{4}}+\frac{x^3}{4\sqrt{5}}+\frac{x^4}{5\sqrt{6}}+\ldots,\quad x > 0$ (6,6)

6. (a) Prove that every absolute convergent series converges. Examine the alternating series $1-\frac{1}{2}+\frac{1}{3}-\frac{1}{4}+\ldots$ for absolute convergence.

(b) Show that the function f defined by

$f(x) = |x| + |x - 1|$ is continuous but not derivable at $x = 0$, $x = 1$.

(6,6)

7. (a) State and prove Rolle's Theorem.

(b) Obtain Maclaurin's series expansion of sinx.

(c) Find the value of 'a' that will make

$\lim_{x\to 0}\left(\frac{\sin 3x - a\sin x}{x^3}\right)$ finite and hence find the limit. (5,4,3)

(100)