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Sr. No. of Question Paper : 2078

GC-3

Your Roll No.....

Unique Paper Code : 32371303

Name of the Paper : Mathematical Analysis

Name of the Course : B.Sc. (H) STATISTICS (CBCS)

Semester : III

Duration : 3 Hours

Maximum Marks : 75

Instructions for Candidates

1. Write your Roll No. on the top immediately on the receipt of this question paper.
2. Attempt in all 4 Questions from **Section I** and 3 Questions from **Section II**.
3. Question numbers 1 and 6 are compulsory.
4. Use separate answer books for **Section I** and **Section II**.

Section – I

1. (a) Write the Supremum and Infimum of the following :

$$S = \left\{ 1 - \frac{1}{3^n}, n \in \mathbb{N} \right\}$$

- (b) (i) Give an example of a set (other than \mathbb{R} and \emptyset) which is a neighbourhood of each of its points.
(ii) Give an example of a closed set which is not an interval.
- (c) State whether the following statements are True or False
(i) A finite set has no limit point.

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(ii) A sequence can converge to more than one limit.

- (d) Assuming that $n^{\frac{1}{n}} \rightarrow 1$ as $n \rightarrow \infty$. Show by applying Cauchy's n^{th} root test that the series $\sum_{n=1}^{\infty} (n^{\frac{1}{n}} - 1)^n$ converges.
- (e) Give the geometrical interpretation of Lagrange's Mean Value Theorem. (2×5)
2. (a) Show that the set of rational numbers is not order complete.
- (b) Define neighbourhood of a point. If M and N are neighbourhoods of a point p , then prove that $M \cap N$ is also a neighbourhood of p . (6,6)
3. (a) Prove that $\lim_{n \rightarrow \infty} \left(1 + \frac{1}{n}\right)^n$ exists and lies between 2 and 3.
- (b) Show that every Cauchy sequence is bounded. (6,6)
4. (a) Test the series $1 + \frac{x^2}{2} + \frac{x^4}{4} + \frac{x^6}{6} + \dots$ for convergence for all positive values of x .
- (b) Let f be the function defined on $[0, 1]$ by setting
- $$f(x) = \frac{1}{2^n}, \text{ when } \frac{1}{2^{n+1}} \leq x \leq \frac{1}{2^n}, \quad n = 0, 1, 2, 3, \dots$$
- $$f(0) = 0$$
- Examine the continuity of the function at the points $x = \frac{1}{2}, \frac{1}{2^2}, \dots, \frac{1}{2^n}, 1$. (6,6)
5. (a) Show that there is no real number k for which the equation $x^3 - 3x + k = 0$ has two distinct roots in $[0, 1]$.
- (b) Obtain Maclaurin's series expansion of e^x . (6,6)

Section – II

6. (a) If u_x be a function whose differences when the increment of x is unity are denoted by $\delta u_x, \delta^2 u_x, \dots$ and $\Delta u_x, \Delta^2 u_x, \dots$ when the increment of x is n , then what is the relation between δ and Δ .
- (b) What is the value of $\Delta^4 f(x)$?, where $f(x) = x^3 + 4x^2 + 3$.
- (c) Represent $f(x) = x^3 + 4x^2$ in terms of factorial notations.
- (d) State properties of Cote's numbers.

(e) $\frac{1}{2}\delta^2 + \delta\sqrt{\left(1 + \frac{1}{4}\delta^2\right)} \equiv \dots$, where δ is a central difference operator. (1×5)

7. (a) Show that

$$y_4 = y_0 + 4\Delta y_0 + 6\Delta^2 y_0 + 10\Delta^3 y_0, \text{ as far as third differences.}$$

- (b) Find the relation between α, β, γ in order that $\alpha + \beta x + \gamma x^2$ may be expressible in one term in the factorial notation.

- (c) Show that

$$u_{2n} - \binom{n}{1} 2^1 u_{2n-1} + \binom{n}{2} 2^2 u_{2n-2} \dots \dots \dots + (-2)^n u_n = (-1)^n (c - 2an),$$

where $u_x = ax^2 + bx + c$. (4×3)

8. (a) Obtain Lagrange's interpolation formula in the form

$$f(x) = \sum_{r=1}^n \frac{L(x).f(x_r)}{(x-x_r).L'(x_r)} = \sum_{r=1}^n L_r(x).f(x_r),$$

where $L(x) = (x-x_1)(x-x_2) \dots (x-x_n)$.

(b) Prove that

$$\Delta^r f_i = \delta^r f_{i+\frac{r}{2}} = \nabla^r f_{i+r} = r! h^r f[x_i, x_{i+1}, \dots, x_{i+r}],$$

where $f_i = f(x_i)$. (6,6)

9. (a) Prove that, if $f(x)$ is a continuous function whose fifth differences are constant,

$$\text{then } \int_{-1}^1 f(x) dx = \frac{8}{9} f(0) + \frac{5}{9} \{f(\sqrt{0.6}) + f(-\sqrt{0.6})\}.$$

(b) Solve any **two** of the following difference equations :

(i) $u_{x+2} + a^2 u_x = \cos ax$

(ii) $u_{x+2} - 7u_{x+1} + 10u_x = 12.4^x$

(iii) $u_{x+4} + u_x = 0$ (6,6)