[This question paper contains 4 printed pages.]

Sr. No. of Question Paper : 2076 **GC-3** **Your Roll No................**

Unique Paper Code : 32371301

Name of the Paper : Sampling Distribution

Name of the Course : **B.Sc. (H) STATISTICS – CBCS**

Semester : III

Duration : 3 Hours Maximum Marks : 75

Instructions for Candidates

1. Write your Roll No. on the top immediately on the receipt of this question paper.
2. Question No. 1 is compulsory.
3. Attempt **Six** questions in all by selecting at least **two** questions from each Section.

1. Attempt any **five** parts :

(a) Define convergence in probability and convergence with probability one and state their relations.

(b) Let $X_1, X_2,...,X_n$ be a random sample of size n from a uniform population with p.d.f.

$$f(x) = \begin{cases} 1, & \text{if } 0 \le x \le 1 \\ 0, & \text{otherwise} \end{cases}$$

Obtain the pdf of $X_{(r)}$.

(c) Discuss null hypothesis, critical region and level of significance with examples.

(d) If $X \sim \chi^2_n$; then prove that $\frac{X-n}{\sqrt{2n}}$ is a N(0,1) variate for large n.

(e) If the variable t has Student's distribution with 2 degrees of freedom, then find $P\left(-\sqrt{2} \le t \le \sqrt{2}\right)$.

(f) Let X_1 and X_2 be independent random variables with density law $f(x) = e^{-x}$, $x \ge 0$, then show that $Z = X_1/X_2$ has F-distribution.

(g) If $X \sim U[0,1]$, then show that $-2 \log X \sim \chi^2_2$. (5×3)

Section A

2. (a) Let g(x) be a non-negative function of a r.v. X. Then show that for every k>0, we have

$$P(g(x) \ge k) \le E(g(x))/k.$$

Hence, obtain Chebychev's inequality. Use it to prove that in 2000 throws of a coin the probability that the number of heads lies between 900 and 1100 is at least 19/20.

(b) Let $X_1, X_2, \ldots, X_n$ be iid random variables and $S_n = X_1 + X_2 + \ldots + X_n$. Obtain the limiting distribution of S_n when n tends to ∞. (6,6)

3. (a) Let $\{X_n\}$ be a sequence of mutually independent random variables such that

$X_n = \pm 1$ with probability $\dfrac{1-2^{-n}}{2}$ and $X_n = \pm 2^{-n}$ with probability 2^{-n-1}. Examine whether the weak law of large numbers can be applied to the sequence $\{X_n\}$.

(b) Show that in odd samples of size n from U[0, 1] population, the mean and variance of the distribution of median are ½ and $1/[4(n+2)]$ respectively. (6,6)

4. (a) Discuss the test of significance for the difference of two means for large sample sizes. Also obtain the confidence interval for it.

(b) Explain the term 'standard error and sampling distribution'. Show that in a series of n independent Bernoulli trials with constant probability of success P, the standard error of the proportion of success

is $\sqrt{PQ/n}$, where $Q = 1-P$. (6,6)

Section B

5. (a) If $X_1, X_2, \ldots, X_n$ are independent random variables with continuous distribution functions $F_1, F_2, \ldots, F_n$ respectively, then show that

$$-2\log\left[F_1(x_1)F_2(x_2)\ldots F_n(x_n)\right] \sim \chi^2_{2n}.$$

(b) Le $P_x = \frac{1}{2^{n/2}\Gamma(n/2)}\int_0^x w^{\frac{n-2}{2}} e^{-w}\,dw,\ x > 0$. Show that $x < \frac{n}{1-P_x}$.

(c) For the t- distribution with n d.f., prove that

$$\mu_{2r} = \frac{n(2r-1)}{(n-2r)}\mu_{2r-2},\ n > 2r.$$ (4,4,4)

6. (a) Prove that ns^2/σ^2 is distributed as chi-square with (n-1) d.f. where s^2 and σ^2 are the variances of sample (of size n) and the population respectively.

(b) Define F-distribution. For F-distribution with n_1, n_2 d.f. show that the mean is independent of n_1 and mode lies between 0 and 1. (6,6)

7. (a) If $X \sim F_{m,n}$ then show that $U = \frac{mX}{n+mX} \sim \beta_1\left(\frac{m}{2}, \frac{n}{2}\right)$.

(b) If $\bar{X}$ and S^2 be the usual sample mean and sample variance based on a random sample of n observations from $N(\mu, \sigma^2)$ and if $T = \frac{\bar{X}-\mu}{S/\sqrt{n}}$, then prove that

$$Cov(\bar{X},T) = \frac{\sigma\sqrt{n-1}\,\Gamma\left(n-2\right)/2}{\sqrt{2n}\,\Gamma\left(n-1\right)/2}, \quad S^2 = (n-1)^{-1}\sum_{i=1}^{n}\left(X_i - \bar{X}\right)^2$$ (6,6)

8. (a) What is a contingency table ? Describe how the χ^2 distribution may be used to test whether the two attributes are independent.

(b) Let $x_1, x_2, \ldots, x_n$ be independent observations from the normal universe with mean μ and variance σ^2 and let $\bar{x}$ and s^2 be the sample mean and sum of the squares of the deviations from the mean respectively. Let x′ be one more observation independent of previous ones. Obtain the distribution of

$$U = \frac{x' - \bar{x}}{s}\sqrt{\frac{n(n-1)}{n+1}}.$$

(c) Prove that if $X \sim F_{m,n}$ and $Y \sim F_{n,m}$, then for every $a>0$, $P(X \leq a) + P(Y \leq 1/a) = 1$. (4,4,4)