This question paper contains **8+1** printed pages]

Roll No. | | | | | | | | | | | |

S. No. of Question Paper : **1864**

Unique Paper Code : **237404** **C**

Name of the Paper : **STHT-403 (Operational Research)**

Name of the Course : **B.Sc. (Hons.) (Statistics)**

Semester : **IV**

Duration : **3 Hours** Maximum Marks : **75**

(Write your Roll No. on the top immediately on receipt of this question paper.)

Attempt any *five* questions. Use of simple calculator is allowed.

SECTION A

1. (*a*) Solve the following L.P.P. :

$$\text{Max. } Z = 2x_1 + 4x_2 + 3x_3$$

subject to constraints :

$$x_1 + 4x_2 + 3x_3 \leq 240$$

$$2x_1 + x_2 + 5x_3 \leq 300$$

$$x_1, x_2, x_3 \geq 0.$$

P.T.O.

(*b*) Suppose the vector $b' = [240,300]$ is changed to $b' = [300,400]$. Find the effect of this change on the optimality of the solution.

(*c*) Write dual of the above problem and get its solution from the optimal primal table. 7,5,3

2. (*a*) Solve the following L.P.P. :

$$\text{Max. } Z = 3x_1 + 2x_2 + x_3$$

subject to constraints :

$$3x_1 + 5x_2 + x_3 = 12$$

$$3x_1 + 4x_2 = 11$$

x_1 unrestricted in sign

$$x_2, x_3 \geq 0.$$

(*b*) The data collected on running cost of a machine along with its resale value in subsequent years is given ahead; the purchasing cost

of machine is Rs. 60,000. Determine the optimum period for replacement of the machine.

Year	**1st**	**2nd**	**3rd**	**4th**	**5th**
Resale value (Rs.)	42,000	30,000	20,400	14,400	9,650
Cost of spare (Rs.)	4,000	4,270	4,880	5,700	6,800
Cost of labour (Rs.)	14,000	16,000	18,000	21,000	25,000

9,6

3. (*a*) Consider the following L.P.P. :

$$\text{Max. } Z = 3x_1 + 2x_2 + 5x_3$$

subject to constraints :

$$x_1 + 2x_2 + x_3 \leq a_1$$

$$3x_1 + 2x_3 \leq a_2$$

$$x_1 + 4x_2 \leq a_3$$

$$x_1, x_2, x_3 \geq 0,$$

P.T.O.

where a_1, a_2 and a_3 are constants. For specific values of a_1, a_2 and a_3 the optimal solution is :

Basic Variable	Solution	x_1	x_2	x_3	x_4	x_5	x_6
x_2	100	b_1	1	0	$\frac{1}{2}$	$-\frac{1}{4}$	0
x_3	c_3	b_2	0	1	0	$\frac{1}{2}$	0
x_6	20	b_3	0	0	–2	1	1
Z	1350	4	0	0	c_1	c_2	0

Evaluate the following :

(*i*) The values of a_1, a_2 and a_3 that yield the given optimal solution.

(*ii*) The values of b_1, b_2, b_3 and c_1, c_2, c_3 in the above given optimal table.

(*iii*) The optimal dual solution.

(*b*) The following table gives the cost of transporting material from supply points A, B, C and D to demand points E, F, G, H and J :

		To				
		E	F	G	H	J
From	A	8	10	12	17	15
	B	15	13	18	11	9
	C	14	20	6	10	13
	D	13	19	7	6	12

The present allocation is as follows :

A to E → 90, A to F → 10, B to F → 150, C to F → 10, C to G → 50

C to J → 120, D to H → 210, D to J → 70.

Check, if the allocation is optimum. If not, find the optimum allocation. 9,6

P.T.O.

SECTION B

4. (*a*) The captain of a cricket team has to allot five middle batting positions to five batsmen. The average runs scored by each batsman at these positions are as follows :

Batting Positions

Batsman	I	II	III	IV	V
P	40	40	35	25	50
Q	42	30	16	25	27
R	50	48	40	60	50
S	20	19	20	18	25
T	58	60	59	55	53

(*i*) Find the assignment of batsmen to positions, which would give the maximum number of runs.

(*ii*) What would be the effect if another batsman 'U' plays with the following average runs in batting positions ?

Batting Positions	**Average runs**
I	45
II	52
III	38
IV	50
V	49

(*b*) A businessman deals in a perishable commodity, the daily demand and supply of which are random variables. The past 500 days data show the following :

Supply		**Demand**	
Available (kg)	**No. of days**	**Demand** (kg)	**No. of days**
10	40	10	50
20	50	20	110
30	190	30	200
40	150	40	100
50	70	50	40

The businessman buys the commodity at Rs. 20 per kg and sells at Rs. 30 per kg. If anyone of the commodity remains at the end of day, it has no saleable value and is a dead loss. Moreover, the loss of any unsatisfied demand is Rs. 8 per kg. Given the following random numbers, simulate six days sale using random numbers in pairs alternately to simulate supply and demand :

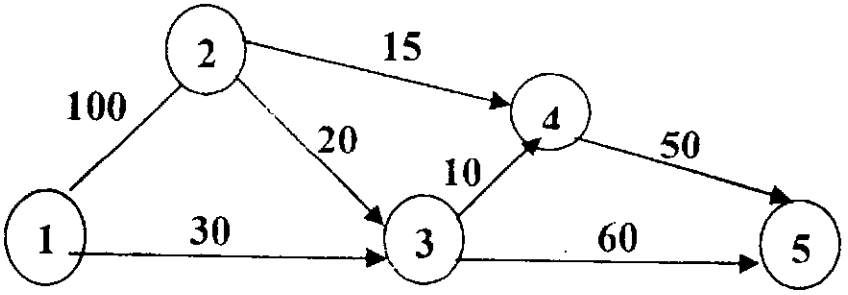
(31, 18); (63, 84); (15, 79); (07, 32); (43, 75); (81, 27). 9,6

5. (*a*) Solve the following two-person zero-sum game :

Player B

Player A				
	1	2	3	−1
	2	2	1	5
	3	1	0	−2
	4	3	2	6

(*b*) Consider the following network which gives the permissible routes and their lengths in miles between city 1 (node 1) and four cities (nodes 2 to 5). Determine the shortest routes and the distances from city 1 (node 1) to each of the remaining four cities.



9,6

6. (*a*) How can you detect it in course of simplex computations, if a linear programming problem possesses :

(*i*) Unbounded solution;

(*ii*) Infeasible solution;

(*iii*) Multiple solutions.

(*b*) Discuss two-person zero-sum game and formulate it as a linear programming problem. 9,6