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Roll No. | | | | | | | | | | | |

S. No. of Question Paper : **1861**

Unique Paper Code : **237401** **C**

Name of the Paper : **STHT-401 (Numerical Analysis)**

Name of the Course : **B.Sc. (Hons.) (Statistics)**

Semester : **IV**

Duration : **3 Hours** Maximum Marks : **75**

(Write your Roll No. on the top immediately on receipt of this question paper.)

Attempt *five* questions in all selecting at least *two* questions from each Section.

SECTION I

1. (*a*) Show that the nth divided differences of a polynomial of the nth degree are constant. 5

(*b*) Show that :

$$\Delta^n O^m = n(\Delta^{n-1} O^{m-1} + \Delta^n O^{m-1})$$

where n and m are positive integers. 5

P.T.O.

(c) Use the method of separation of symbols to prove the following identity :

$$U_0 - U_1 + U_2 - U_3 + ... = \frac{1}{2}U_0 - \frac{1}{4}\Delta U_0 + \frac{1}{8}\Delta^2 U_0 - \frac{1}{16}\Delta^3 U_0 + ... \quad 5$$

2. (a) Obtain Newton's divided difference formula and hence derive Newton's forward difference formula. 8

(b) The values of $f(x)$ are given at a, b and c. Show that under certain conditions, the maximum or minimum is attained at : 7

$$x = \frac{\Sigma(b^2 - c^2) f(a)}{2\Sigma(b - c) f(a)}.$$

3. (a) Obtain Stirling's central difference interpolation formula. 8

(b) Establish the following relation : 7

$$\Delta^r f_i = \delta^r f_{i+\frac{r}{2}} = \nabla^r f_{i+r} = r! h^r f[x_i, x_{i+1}, ...x_{i+r}].$$

4. (a) What is the purpose of inverse interpolation ? Discuss the method of successive approximation or iteration for inverse interpolation. 8

(*b*) If U_x be a function whose differences when the increment of x is unity are denoted by δU_x, $\delta^2 U_x$,..... and by ΔU_x, $\Delta^2 U_x$, ... when the increment of x is n, then if $\delta^2 U_x$, $\delta^2 U_{x+1}$, $\delta^2 U_{x+2}$, ... are in G.P. with common ratio q, show that : 7

$$\frac{\Delta U_x - n\delta U_x}{(q^n - 1) - n(q - 1)} = \frac{\delta^2 U_x}{(q - 1)^2}.$$

SECTION II

5. (*a*) State and prove Euler-Maclaurin summation formula. 7

(*b*) If y_x is a polynomial in x of the third degree, find an expression for $\int_0^t y_x dx$ in terms of y_0, y_1, y_2 and y_3. Use this result to find the value of $\int_1^2 y_x dx$. 8

6. (*a*) Sum the following series to n terms : 8

2, 12, 36, 80, 150, 252, ...

(*b*) Show that $\Delta(x!) = x(x!)$ and hence find the value of 7

$$\sum_1^n (x^2 + 1)(x!).$$

P.T.O.

7. Solve any *three* of the following difference equations :

(*i*) $U_{x+2} - 4U_{x+1} + 4U_x = 2^x$

(*ii*) $U_{x+2} - 2U_{x+1} + U_x = x^{(3)} + x(-1)^x$

(*iii*) $U_{x+1} - pa^{2x}U_x = qa^{x2}$

(*iv*) $U_{x+2} + a^2U_x = \cos ax.$ 5,5,5