This question paper contains 4+2 printed pages]

Roll No. | | | | | | | | | | | |

S. No. of Question Paper : **1862**

Unique Paper Code : **237402** **C**

Name of the Paper : **STHT-402 (Probability and Statistical Methods—IV)**

Name of the Course : **B.Sc. (Hons.) (Statistics)**

Semester : **IV**

Duration : **3 Hours** Maximum Marks : **75**

(Write your Roll No. on the top immediately on receipt of this question paper.)

Attempt *five* questions in all, selecting *two* questions from Section I and *three* questions from Section II.

SECTION I

1. (*a*) For the exponential density function $f(x) = e^{-x}$, $x \geq 0$, find the probability density function of the range R in a random sample of size n and show that :

$$E(R) = 1 + \frac{1}{2} + \frac{1}{3} + \dots + \frac{1}{n-1}.$$

(*b*) Let $X_1, X_2, \ldots, X_n$ be a random sample of size n from a population having continuous distribution function $F(x)$. Define order statistics of rank k, $1 \leq k \leq n$. Find its distribution function. Show that for the rectangular distribution :

$$f(x) = \frac{1}{\theta_2}, \theta_1 - \frac{\theta_2}{2} \leq x \leq \theta_1 + \frac{\theta_2}{2},$$

$$E\left[\frac{X_{(r)} - \theta_1}{\theta_2}\right] = \frac{r}{n+1} - \frac{1}{2}.$$ 7,8

2. (*a*) Prove that the mean and variance of a random sample drawn from a normal population are independently distributed. Also write their probability density functions.

(*b*) Let X be distributed as $\chi^2_{(n)}$. Show that

$$x \leq \frac{n}{1 - F(x)},$$

where $F(x) = P[X \leq x]$.

(c) Find the p.d.f. of $\chi_{(n)} = +\sqrt{\chi^2_{(n)}}$, where $\chi^2_{(n)}$ is a Chi-square variate with n degrees of freedom. Also show that :

$$\mu'_r = E(\chi^r_{(n)}) = 2^{r/2}\frac{\Gamma[(n+r)/2]}{\Gamma[n/2]}.$$

Hence, establish that for large n

$$E(\chi^2_{(n)}) = \left(E(\chi_{(n)})\right)^2.$$ 7,3,5

3. (a) If X is Poisson (λ) and χ^2 is a Chi-square variate with $2k$ degrees of freedom. Prove that for all positive integer k,

$$P[X \leq k - 1] = P[\chi^2 > 2\lambda]$$

(b) Describe the test for testing the significance of the homogeneity of several sample correlation coefficients from a bivariate normal population.

(c) Let $X_1, X_2, ..., X_n$ be a random sample from a population with continuous density. Show that $Y_1 = \min(X_1, X_2, ..., X_n)$ is exponential with parameter $n\lambda$ if and only if each X_i is exponential with parameter λ. 5,5,5

Section II

4. (*a*) Define sampling distribution and standard error of a statistic. Obtain standard error of a sample mean in a random sample of size n from a large population with standard deviation σ.

(*b*) What is meant by a statistical hypothesis ? What are the two types of errors involved in testing of a statistical hypothesis ? Point out the difference between one-tailed and two-tailed tests.

(*c*) Let p_1 and p_2 be the proportions obtained from samples of size n_1 and n_2 respectively from two populations. Suggest an unbiased estimate of $(P_1 - P_2)$, where P_1 and P_2 are the corresponding population proportions. Also obtain its sampling distribution assuming n_1 and n_2 to be large. Hence explain how to test the hypothesis $P_1 = P_2$. 5,5,5

5. (a) Show that for t-distribution with n degrees of freedom, mean deviation about mean is given by

$$\frac{\sqrt{n}\,\Gamma[(n-1)/2]}{\sqrt{\pi}\,\Gamma[n/2]}.$$

(b) If (X_i, Y_i), $i = 1, 2, \ldots, n$ be a random sample of size n from an uncorrelated bivariate normal population, derive the distribution of sample correlation coefficient r. Hence, obtain the p.d.f. of $u = \frac{r\sqrt{n-2}}{\sqrt{(1-r^2)}}$ and identify it. 5,10

6. (a) Prove that if $n_1 = n_2$, the median of F-distribution is at $F = 1$ and that the quartiles Q_1 and Q_3 satisfy the condition $Q_1 \cdot Q_3 = 1$.

(b) Let $X \sim F_{(2,n)}$, $n \geq 2$. Show that :

$$P[X \geq k] = \left[1 + \frac{2k}{n}\right]^{-n/2}$$

Deduce the significance level of F-distribution corresponding to the significance level of probability p.

(c) Let $X \sim F_{(n,\,m)}$. Obtain the p.d.f. of $Y = nX$, when m is large. 5,5,5

7. Write short notes on any *three* of the following :

(*i*) Mode of F-distribution and its characteristics;

(*ii*) Applications of *t*-distribution;

(*iii*) Joint p.d.f. of *r*th and *s*th order statistics.

(*iv*) Test of significance for testing difference of means for large samples. 15