[This question paper contains 2 printed pages.]

Sr. No. of Question Paper : 1205 **E** **Your Roll No.....**

Unique Paper Code : 237401

Name of the Course : **B.Sc. (Hons.) Statistics**

Name of the Paper : Numerical Analysis (STHT–401)

Semester : IV

Duration : 3 Hours Maximum Marks : 75

Instructions for Candidates

1. Write your Roll No. on the top immediately on receipt of this question paper.
2. Attempt **5** questions in all, selecting atleast **2** question from each Section.

SECTION I

1. (a) Prove that the divided differences are symmetrical in all their arguments.

   (b) Represent the function $f(x) = 2x^4 - 12x^3 + 24x^2 - 30x + 9$, and its successive differences in factorial notation.

   (c) Prove that

$$u_0 + \frac{u_1 x}{1!} + \frac{u_2 x^2}{2!} + \ldots = e^x\left[u_0 + x\Delta u_0 + \frac{x^2}{2!}\Delta^2 u_0 + \ldots\right] \qquad (5,5,5)$$

2. (a) Derive Lagrange's interpolation formula. Show that sum of the Lagrangian coefficients is unity.

   (b) Show that for the interpolation of f(x) relative to 0, α, 1 Lagrange's formula gives :

$$f(x) \cong \left[1 - \frac{x(x-\alpha)}{1-\alpha}\right] f(0) + \frac{x(1-x)}{1-\alpha} \times \frac{f(\alpha) - f(0)}{\alpha} + \frac{x(x-\alpha)}{1-\alpha} \times f(1).$$

   Also show that if $\alpha \to 0$, it reduces to :

$$f(x) = (1 - x^2)f(0) + x(1 - x)f'(0) + x^2 f(1) \qquad (8,7)$$

P.T.O.

3. (a) By means of usual notations Δ, ∇, and δ, establish the following relation :

$$\Delta^r f_i = \delta^r f_{i+\frac{r}{2}} = \nabla^r f_{i+r} = r!h^r f\left[x_i, x_{i+1}, \text{---} -, x_{i+r}\right], \text{ where } f_i = f(x_i)$$

(b) Prove that

$$\frac{\Delta^n 0^m}{n!} = \frac{n\Delta^n 0^{m-1}}{n!} + \frac{\Delta^{n-1} 0^{m-1}}{(n-1)!}$$

where n and m are positive integers. (9.6)

4. (a) What is the purpose of inverse interpolation ? Discuss the method of reversion of series for inverse interpolation.

(b) Estimate the missing term in the following table :

x	1	2	3	4	5	6	7
y_x	2	4	8	–	32	64	128

Explain why the result differs from 2^4. (8.7)

SECTION II

5. (a) Obtain Newton-Cotes integration formula. Hence derive Simpson's $\frac{1}{3}$ rd rule.

(b) If f(x) is a function whose fifth differences are constant, then $\int_{-1}^{1} f(x)dx$ can be expressed in the form $pf(-\alpha) + qf(0) + pf(\alpha)$.

Find the values of p, q and α. (9,6)

6. (a) Show that $\Delta(x!) = x(x!)$ and hence find the sum of n terms of the series $1 + 3(2!) + 7(3!) + 13(4!) + 21(5!) + \ldots$

(b) Find the general term and the sum upto n terms of the series

1, 2, 4, 8, 17, 40, 104, ...

7. Solve any **three** of the following difference equations :

(i) $u_{x+2} - 7u_{x+1} + 10u_x = 12.4^x$

(ii) $u_{x+1} - au_x = \sin bx$

(iii) $u_{x+2} - 7u_{x+1} + 8u_x = x^{(2)}.2^x$

(iv) $u_{x+1} = 2u_x\sqrt{1-u_x^2}$ (5.5.5)

(100)