[This question paper contains 4 printed pages.]

Sr. No. of Question Paper : 1206 **E** **Your Roll No...............**

Unique Paper Code : 237402

Name of the Course : **B.Sc. (Hons.) Statistics**

Name of the Paper : Probability and Statistical Methods – IV (STHT–402)

Semester : IV

Duration : 3 Hours Maximum Marks : 75

Instructions for Candidates

1. Write your Roll No. on the top immediately on receipt of this question paper.
2. Attempt **five** questions in all selecting atleast **two** from each Section I and **three** from Section II.

SECTION I

1. (a) Obtain the cumulative distribution function and hence the p.d.f. of the smallest sample observation $X_{(1)}$ in a random sample of size n from a continuous population with cumulative distribution function F(x). Show that for a random sample of size 2 from normal population

$$N\left(0,\sigma^2\right),\ E\left(X_{(1)}\right) = {-\sigma}/{\sqrt{\pi}}.$$

(b) Given a random sample of size n form the exponential distribution :

$$f(x) = \alpha e^{-\alpha x}, \quad \alpha > 0,\ x \geq 0.$$

(i) Show that $X_{(r)}$ and $W_{rs} = X_{(s)} - X_{(r)}$, r < s, are independent.

(ii) Find the distribution of $W_1 = X_{(r+1)} - X_{(r)}$. (8,7)

P.T.O.

2. (a) Prove that the mean and variance of a random sample drawn from a normal population are independently distributed. Also write their probability density functions.

(b) Find the p.d.f. of $\chi_{(n)} = +\sqrt{\chi^2_{(n)}}$, where $\chi^2_{(n)}$ is a chi-square variate with n degrees of freedom. Also show that $\mu'_r = E\left(\chi^r_{(n)}\right) = 2^{r/2} \frac{\Gamma[(n+r)/2]}{\Gamma[n/2]}$.

(8,7)

3. (a) If X is distributed as a chi-square variate with n degrees of freedom then prove that for large n

$$\sqrt{2X} \sim N\left(\sqrt{2n}, 1\right).$$

(b) If $X_1, X_2, ..., X_n$ are i.i.d. exponential variates with parameter λ then prove that $2\lambda \Sigma X_i \sim \chi^2_{(2n)}$.

(c) Let $X_1, X_2, ..., X_n$ be a random sample of size n from Uniform (0, 1) distribution. Show that

$$Y_1 = X_{(1)}/X_{(2)},\ Y_2 = X_{(2)}/X_{(3)}, ..., Y_n = X_{(n-1)}/X_{(n)} \text{ and } Y_n = X_{(n)}$$

are independently distributed. Also identify their distributions. (5,5,5)

SECTION II

4. (a) Define the terms parameter, statistic, sampling distribution, statistical hypothesis, type I and type II errors.

(b) Discuss the test of significance for the difference between two standard deviations for large samples.

(c) Obtain standard error of a sample mean in a random sample of size n from a large population with variance σ^2. (6,5,4)

5. (a) Show that for t-distribution with n degree of freedom mean deviation about mean is given by

$$\frac{\sqrt{n}\ \Gamma[(n-1)/2]}{\sqrt{\pi}\ \Gamma[n/2]}.$$

(b) Let X_1 and X_2 be two independent normal variates each with the same mean μ and variance σ^2. Obtain the distribution of $Y = \frac{(X_1 + X_2 - 2)}{\sqrt{|X_1 - X_2|^2}}$ and identify it.

(c) Explain, stating clearly the assumptions involved, the t-test for testing the significance of the difference between the two sample means. (5,5,5)

6. (a) Prove that if $n_1 = n_2$, the median of F-distribution with n_1 and n_2 degrees of freedom is at F = 1 and that the quartiles Q_1 and. Q_3 satisfy the condition $Q_1.Q_3 = 1$.

(b) If $X \sim F_{(2,n)}$, $n \geq 2$ then show that

$$P[F \geq 2] = \left[1 + \frac{2k}{n}\right]^{-\frac{n}{2}}$$

Deduce the significance level of F corresponding to the significance level of probability p.

(c) If $X \sim F_{(n,m)}$ then obtain the p.d.f. of Y = nX, when m is large. (5,5,5)

7. Write short notes on any **three** of the following :

   (i) Chi-square test for goodness of fit.

   (ii) Applications of F-distribution.

   (iii) Paired t-test.

   (iv) Test of significance for difference of means from two different normal populations for large sample sizes. (5,5,5)