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Roll No. | | | | | | | | | | | |

S. No. of Question Paper : **2372**

Unique Paper Code : **2371403** **F-4**

Name of the Paper : **Probability and Statistical Methods–IV**

Name of the Course : **B.Sc. (Hons.) Statistics**

Semester : **IV**

Duration : **3 Hours** Maximum Marks : **75**

(Write your Roll No. on the top immediately on receipt of this question paper.)

Attempt *five* questions in all, selecting *two* from section I and *three* from section II.

Attempt all parts of a question in continuation.

Section I

1. (*a*) Define *r*th order statistics $X_{(r)}$. Obtain the joint p.d.f. of $X_{(r)}$ and $X_{(s)}$, $r < s$, in a random sample of size n from a population with continuous distribution function F (.). Hence deduce the p.d.f. of sample range $W = X_{(n)} - X_{(1)}$.

(*b*) Let $X_1, X_2, \ldots\ldots X_n$ be a random sample from a population with continuous density function. Show that :

$$Y_{(1)} = \min(X_1, X_2, \ldots\ldots, X_n)$$

is exponential with parameter $n\lambda$ if and only if each X_i is exponential λ. 10,5

P.T.O.

2. (*a*) Let $X_1, X_2, \ldots\ldots X_n$ be a random sample from $N(\mu, \sigma^2)$ and $\bar{X}$ and S^2 the sample mean and sum of the squares of the deviations from the mean respectively. Show that $\bar{X}$ and S^2 are independent also find their distributions.

(*b*) If $X_1, X_2, \ldots\ldots X_n$ are i.i.d. exponential variates with parameter λ, prove that :

$$2\lambda \sum X_i \sim \chi^2_{(2n)}.$$

10.5

3. (*a*) Find the p.d.f. of $\chi_n = +\sqrt{\chi_n^2}$, where χ_n^2 is a χ^2 variate with *n* degrees of freedom and show that :

$$\mu_r' = E\chi_n^r = 2^{r/2} \frac{\Gamma[(n+r)/2]}{\Gamma(n/2)}.$$

Hence establish that for large *n* $E(\chi_n^2) = [E(\chi_n)]^2$.

(*b*) Derive Brandt and Snedecor formula for $2 \times k$ contingency table. 10.5

Section II

4. (*a*) Obtain the distribution of sample correlation coefficient *r* when the population correlation coefficient $\rho = 0$. Hence deduce that :

$$\frac{r}{\sqrt{(1-r^2)}}\sqrt{(n-2)}$$

follows Student's *t*-distribution with $(n - 2)$ degrees of freedom.

(*b*) If $X \sim t_{(n)}$, then show that :

$$(n - \frac{1}{2}) \log (1+\frac{X^2}{n}) \sim \chi^2_{(1)}$$

for large *n*. 10.5

5. (*a*) Prove that if $n_1 = n_2$, the median of $F(n_1, n_2)$ distribution is at F = 1 and that the quartiles Q_1 and Q_3 satisfy the condition $Q_1Q_3 = 1$.

(*b*) If $X \sim F(n_1, n_2)$, then show that :

$$U = \frac{n_1 X}{n_2 + n_1 X} \sim \beta_1\left(\frac{n_1}{2}, \frac{n_2}{2}\right).$$

(*c*) If $X \sim F_{(2,n)}$, $n \geq 2$, then show that :

$$P[F \geq 2] = \left[1 + \frac{2k}{n}\right]^{-\frac{n}{2}}.$$

Deduce the significance level of F corresponding to the significant probability P. 5,5,5,

6. (*a*) Define the terms sampling distribution and standard error of a statistic. Show that in the series of *n* independent trials with constant probability P of success, the standard error of the proportion of successes is $\sqrt{\frac{PQ}{n}}$, where Q = 1 − P.

(*b*) If X is distributed as a *t*-statistic, show that X^2 follows F distribution.

(*c*) For the *t*-distribution with *n* degrees of freedom establish the recurrence relation :

$$\mu_{2r} = \frac{n(2r-1)}{(n-2r)}\mu_{2r-2}, n > 2r.$$ 5,5,5

7. Write short notes on any *three* of the following : 5,5,5

(*i*) Yates' correction for continuity

(*ii*) F-test for equality of two population variances

(*iii*) Relation between F and χ^2.

(*iv*) Limiting form of t-distribution.