[This question paper contains 4 printed pages.]

Sr. No. of Question Paper : 2353 **F–4** **Your Roll No...............**

Unique Paper Code : 2371402

Name of the Course : **B.Sc. (Hons.) Statistics**

Name of the Paper : Real Analysis and Numerical Analysis

Semester : IV

Duration : 3 Hours Maximum Marks : 75

Instructions for Candidates

1. Write your Roll No. on the top immediately on receipt of this question paper.
2. Attempt **six** questions in all, selecting **three** from each Section.
3. Attempt all parts of a question in continuation.

SECTION I

1. (a) Define an upper bound and the supremum of a bounded set of real numbers. If S is a non-empty set of real numbers bounded above then a real number s is the supremum of S iff the following two conditions hold :

   (i) $x \leq s$ for all $x \in S$

   (ii) For each positive real number ε, there is a real number $x \in S$ such that $x > s - \varepsilon$.

   (b) Define neighbourhood of a point, open set and limit point of a set. Prove that the intersection of two open sets is open. Does the above result hold for the intersection of an arbitrary family on open sets ? Illustrate your answer by an example. (7½,5)

2. (a) State and prove Monotone Convergence Theorem.

(b) Prove that $\lim_{n\to\infty}\left(1+\frac{1}{n}\right)^n$ exists and lies between 2 and 3. (6,6½)

3. (a) Define sequence of partial sums of a series. Test for convergence of the series

(i) $\frac{1}{\log 2}+\frac{1}{\log 3}+\ldots+\frac{1}{\log n}+\ldots$

(ii) $\frac{1}{2}x+\frac{1.3}{2.4}x^2+\frac{1.3.5}{2.4.6}x^3+\ldots+\frac{1.3\ldots(2n-1)}{2.4\ldots 2n}x^n+\ldots$

for all positive values of x.

(b) Obtain the Maclaurin's series expansion of the function e^x, $x \in R$. (6,6½)

4. (a) Prove that the alternating series

$$u_1 - u_2 + u_3 - u_4 + \ldots\ldots (u_n > 0 \text{ for all } n)$$

whose terms satisfy the conditions

(i) $u_{n+1} \le u_n$, for all n ;

(ii) $u_n \to 0$ as $n \to \infty$,

converges.

(b) Show that the series

$$1 + x + \frac{x^2}{2!} + \ldots \ldots$$

converges absolutely for all values of x. (7½,5)

SECTION II

5. (a) Express the function $2x^3 - 3x^2 + 3x - 10$ and its differences in factorial notation.

(b) Define operators Δ, ∇, E and δ in calculus of finite differences. Establish the relations :

$$\Delta^r f_i = \delta^r f_{i+\frac{r}{z}} = \nabla^r f_{i+r} = r!h^r f\left[x_i, x_{i+1}, \ldots, x_{i+r}\right]$$ (5½,7)

6. (a) Derive Lagrange's interpolation formula.

(b) Show that the limit of Lagrange's interpolation formula for arguments x_0, x_1 and $x_0 + \varepsilon$ as $\varepsilon \to 0$, is

$$f(x) = \frac{(x_1 - x)(x + x_1 - 2x_0)}{(x_1 - x_0)^2} f(x_0) + \frac{(x - x_0)(x_1 - x)}{(x_1 - x_0)} f'(x_0) + \frac{(x - x_0)^2}{(x_1 - x_0)^2} f(x_1)$$

(6,6½)

7. (a) State the Newton-Cotes integration formula and hence obtain Simpson's 1/3rd rule.

(b) Calculate by Simpson's 1/3rd rule, an approximate value of $\int_{-3}^{3} x^4 \, dx$ by taking seven equidistant ordinates. Compare it with exact value.

(6½,6)

8. Solve any **three** of the following difference equations :

(i) $U_{x+2} - 4U_{x+1} + 13U_x = 0$

(ii) $U_{x+2} - 5U_{X+1} + 6U_X = 5^X$

(iii) $U_{x+2} - 4U_x = 9x^2$

(iv) $U_{x+1} - aU_x = \sin bx$ (4,4,4½)