This question paper contains 6 printed pages.]

1433 *Your Roll No. ....*

B.Sc. (Hons.) / III **A**

STATISTICS – Paper XXV

(Econometrics)

(Admissions of 1999 and onwards)

Time : 2 Hours *Maximum Marks : 38*

(Write your Roll No. on the top immediately on receipt of this question paper.)

Answer ***four*** *questions in all, selecting* ***two*** *from each Section.*

SECTION I

1. (a) In the model $Y_t = \beta_0 + \beta_1 X_{1t} + U_t$; $t = 1, 2, ..., n$, assume the first order autoregressive scheme $u_t = fu_{t-1} + \in_t$ where $\in_t$ satisfies the assumptions of classical linear regression model. Show that

(i) $V(U_t) = \sigma_{\in}^2/(1-\rho^2)$ where $\rho_{\in}^2 = \text{var}(\in_t)$;

(ii) What is the covariance between U_t and $U_{tts}; s \neq 0$?

[P.T.O.

(iii) Write the variance-covariance matrix $E(UU')$;

(iv) Hence compute the variance of the ordinary least squares estimate (OLSE) and compare it with the variance of OLSE for classical linear regression model.

(b) Define a forecasting problem. Discuss the procedure of exponential smoothing for a constant process. 6, 3½

2. (a) Discuss multicollinearity ? Also give some of the remedial measures that may be taken to tackle the problem of multicollinearity.

(b) In the regression model

$$Y_t = \beta_0 + \lambda Y_{t-1} + \beta_2 X_t + U_t - \lambda U_{t-1}; 0 < \lambda < 1;$$

$$E(U_t) = 0 \;\; \forall t \text{ and } E(U_t U_{t+s}) = \begin{cases} \sigma^2 & ; \quad s = 0 \\ 0 & ; \quad \forall s \neq 0, \end{cases}$$

discuss the method of computation of Generalised least squares estimator when

(i) λ is known

(ii) λ is unknown.

(c) In studying the movement in the production worker's share in the value added (i.e. labour share) the following models were considered :

Model A : $\gamma_t = \beta_0 + \beta_1 t + U_t$

Model B : $\gamma_t = \alpha_0 + \alpha_1 t + \alpha_2 t^2 + U_t$

Where γ is labour's share and t is time based on annual data for 1949-1964, the following results were obtained for the primary metal industry :

Model A : $\hat{Y}_t = 0.4529 - 0.0041t$; $R^2 = 0.5284$

(-3.9608) ; $d = 0.8252$

Model B : $\hat{Y}_t = 0.4786 - 0.0127t + 0.0005t^2$

(-3.2724) (2.777)

$R^2 = 0.6629$; $d = 1.82$

Figures in parenthesis are t-ratios.

(i) Is there serial correlation in Model A or in Model B ?

(ii) What accounts for serial correlation ?

3½,4,2

3. (a) For a linear model $\underset{\sim}{\gamma} = X\underset{\sim}{\beta} + \underset{\sim}{u}$, derive an appropriate test, to test the hypothesis $H_0 : C'\underset{\sim}{\beta} + r$, where C is an appropriately specified vector of constants and r is a known constant.

(b) Consider the following correlation matrix :

$$R = \begin{array}{c} \\ X_2 \\ X_3 \\ \vdots \\ X_K \end{array} \begin{array}{c} \begin{array}{cccc} X_2 & X_3 & \cdots & X_K \end{array} \\ \begin{bmatrix} 1 & r_{23} & \cdots & r_{2k} \\ r_{32} & 1 & \cdots & r_{3k} \\ \vdots & \vdots & \vdots & \vdots \\ r_{K1} & r_{K2} & \cdots & 1 \end{bmatrix} \end{array}$$

How would you find out from the correlation matrix whether (i) there is perfect collinearity (ii) there is less than perfect collinearity, and (iii) X's are uncorrelated.

(c) Let $\gamma_t = \alpha + \beta X_t + U_t$;

where $E(U_t) = 0; E(U_t^2) = \frac{\sigma^2}{\lambda_t^2}$

and $E(U_t U_s) = 0 \ \forall t \neq s; \sigma^2$ being unknown and $\lambda's$ being known positive numbers. Find the variance of $\hat{\beta}$ (OLSE) and $\hat{b}$ (GLSE) assuming that the variance of the disturbance term is proportional to X *i.e.* $E(U_t^2) = \sigma^2 X_t \ \ t = 1, 2, ..., n$.

3½,2½,3½

SECTION II

4. (a) Define Pareto's law of distribution of income and obtain its Lorenz curve. What is the concentration ratio for v = 1.5 and what is the physical interpretation ?

(b) If $u = X^{\alpha}Y^{\beta}$ is an individual's utility function for two goods, show that his demands for the goods are $X = \frac{\alpha}{\alpha+\beta} \cdot \frac{I}{P_X}$ and $Y = \frac{\beta}{\alpha+\beta} \cdot \frac{I}{P_Y}$

where P_X and P_Y are the fixed prices and 'I' is the individual's fixed income. 5½, 4

5. (a) Define Average Product of Labour (AP_L) and Marginal Product of Labour (MP_L). Also prove that maximum of MP_L curve reaches at a lower level of labour than that of AP_L curve, and it cuts the AP_L curve where AP_L is maximum.

(b) Given the production function

$$X = \sqrt{2HLK - AL^2 - BK^2}$$

show that maximum value of average product of labour is constant $\sqrt{\frac{(H^2 - AB)}{B}}$, which is independent of fixed amount of K used. 5, 4½

6. (a) Define elasticity of substitution. Also find the elasticity of substitution for the production function : $X = \sqrt{2HLK - AL^2 - BK^2}$.

(b) The production function of a firm is given to be

$q = 20 - [(K + L) | LK]$

and prices per unit of K and L are P_L = 20 and P_K = 40. If the unit output price is Rs. 5 determine the amounts of factors demanded, the amount of product produced and the amount of profit.

5, 4½