[This question paper contains 4 printed pages.]

1427 Your Roll No. ....

B.Sc. (Hons.) / III A

STATISTICS – Paper XIX

(Statistical Inference – I)

(Admissions of 1999 and onwards)

Time : 2 Hours *Maximum Marks : 38*

(Write your Roll No. on the top immediately on receipt of this question paper.)

Attempt **four** *questions in all, selecting* **two** *questions from each Section.*

SECTION I

1. (a) State and prove the Invariance Property of consistent estimators.

(b) Let $X_1, X_2, \ldots\ldots, X_n$ be a random sample from uniform population

$$f(x) = \frac{1}{\beta}, \ 0 < x < \beta.$$

Show that $T_1 = \frac{n+1}{n} Y_n$ (where Y_n is the largest

P.T.O.

order statistic of a random sample $X_1, X_2, ..., X_n$) and $T_2 = 2\bar{X}$ are both unbiased estimators for β. Also compare the relative efficiency of T_1 to T_2.

(c) Let T_0 be an MVU estimator and T_1 be an unbiased estimator with efficiency e_θ. Show that no unbiased linear combination of T_0 and T_1 can be an MVU estimator. (3,4,2½)

2. (a) State and prove the Cramer-Rao inequality and explain its significance.

(b) Let $\underset{\sim}{X} \sim N(\theta, r)$, where $r = \frac{1}{\sigma^2}$ is precision and is known. Obtain posterior distribution of θ, when prior distribution of θ is $N(\mu, \tau)$ and τ is the precision. Comment on your result. (5½,4)

3. (a) Let $Y_1 < Y_2 < Y_3 < Y_4 < Y_5$ be the order statistic of the random sample of size 5 from a uniform distribution having p.d.f. $f(x, \theta) = \frac{1}{\theta}$; $0 < x < \theta$, $0 < \theta < \infty$. Show that $2Y_3$ is unbiased for θ. Determine the joint p.d.f. of Y_3 and Y_5 and $E(2Y_3/Y_5) = \varphi(Y_5)$. Also compare the variances of $2Y_3$ and $\varphi(Y_5)$.

(b) Write short notes on

(i) Conjugate Priors

(ii) Bayes rule and the associated Bayes risk (5½,4)

SECTION II

4. (a) Let $X_1, X_2, ..., X_n$ be a random sample from exponential population with p.d.f.

$$f(x, \theta) = e^{-(x-\theta)}, \quad x \geq \theta, \quad -\infty < \theta < \infty.$$

Show that $X_{(1)} = \min(X_1, X_2, ..., X_n)$ is complete sufficient statistic for θ. Find a unique continuous function of this statistic which is best statistic for θ.

(b) In sampling from a Power series distribution with probability function

$$f(x, \theta) = \frac{a_x \theta^x}{\psi(\theta)}; \; x = 0, 1, 2, ...$$

where a_x may be zero for some x. Show that the maximum likelihood estimator of θ is the rod of the equation

$$\bar{x} = \frac{\theta \cdot \psi'(\theta)}{\psi(\theta)} = \mu(\theta),$$

$$\mu(\theta) = E(X).$$ (5½,4)

5. (a) Prove that with probability approaching unity as

$n \to \infty$, the m.l. estimator exists, which converges in probability to true value of θ.

(b) Let $X_1, X_2, \ldots\ldots, X_n$ be a random sample of size n from rectangular distribution with p.d.f.

$$f(x,\theta) = \begin{cases} \frac{1}{\theta}, & 0 \le x \le \theta \\ 0, & \text{elsewhere}, \quad \theta > 0 \end{cases}$$

Let Y be the sample range and ξ be given by

$$\xi^{n-1}[n-(n-1)\xi] = \alpha.$$

Show that Y and $\frac{Y}{\xi}$ are confidence limits for θ with confidence coefficient $(1-\alpha)$. (6,3½)

6. Write short notes on any **three** of the following :

(i) Fisher-Neyman criterion for sufficient statistic

(ii) Rao-Blackwell Theorem

(iii) Method of least squares

(iv) Method of minimum Chi-square (9½)