[This question paper contains 4 printed pages.]

1429 *Your Roll No.* ....

B.Sc. (Hons.)/III **A**

STATISTICS – Paper XXI

(Linear Models)

(Admissions of 1999 and onwards)

Time : 2 Hours *Maximum Marks : 38*

(Write your Roll No. on the top immediately on receipt of this question paper.)

Attempt **Four** *questions in all, selecting* **two** *questions from each Section.*

SECTION I

1. Consider the simple linear regression model $Y = \beta_0 + \beta_1 X + \epsilon$ with usual assumptions.

(a) Show that the sum of the residuals weighted by the corresponding value of the regressor variable is always equal to zero.

(b) Find a $100(1-\alpha)\%$ confidence interval for mean response at a particular value of regressor variable.

(c) Show that $\mathrm{Cov}\left(\hat{\beta}_0, \hat{\beta}_1\right) = -\frac{\bar{X}\sigma^2}{Sxx}$. (3,3½,3)

2. (a) If $\underset{\sim}{Y} \sim Np(\underset{\sim}{\mu}, \Sigma)$, obtain the distribution of

$$(\underset{\sim}{Y} - \underset{\sim}{\mu})' \Sigma^{-1} (\underset{\sim}{Y}, \underset{\sim}{\mu}).$$

(b) Let $\underset{\sim}{Y}' A \underset{\sim}{Y}$ be a quadratic form in $Y_1, Y_2, \ldots, Y_n$ where $Y_i \sim N(0, 1)$; $i = 1, 2, \ldots n$. Prove that if $\underset{\sim}{Y}' A \underset{\sim}{Y}$ is distributed as χ^2 with k d.f. then A is an idempotent matrix of rank k.

(c) Consider $\underset{n\times 1}{\underset{\sim}{Y}} = \underset{n\times K+1}{X} \underset{K+1\times 1}{\underset{\sim}{\beta}} + \underset{n\times 1}{\underset{\sim}{\in}}$ with $R(X) = K + 1 < n$, $E(\underset{\sim}{\in}) = 0$, $V(\underset{\sim}{\in}) = \sigma^2 I$ and $\in$'s are uncorrelated. Find the expected value of $\hat{\beta}' X' X \hat{\beta}$.

(2½,4½,2½)

3. Derive the analysis of variance for balanced two-way cross-classified data with an equal number of observations, say r (>1) in each cell under random effects model. Also discuss the case when r = 1. (9½)

SECTION II

4. (a) Consider three independent random variables Y_1, Y_2 and Y_3 having common variance σ^2 and expectations as given below

$E(Y_1) = \theta_1 + \theta_3$, $E(Y_2) = \theta_1 + \theta_2$, $E(Y_3) = \theta_1 + \theta_3$.

Determine the condition of estimability of the parametric function. Also determine the sum of squares due to error.

(b) Write short notes on :

(i) Orthogonal Columns in X-matrix

(ii) Linear Models. (5½,4)

5. (a) Define multiple linear regression models. 'Which specific regressors seem important in a multiple linear regression model ?' How will you address this question ?

(b) Suppose the postulated model is $E(Y) = \beta_1 X$ but the true model is $E(Y) = \beta_0 + \beta_1 X$. Is the least squares estimator of the slope unbiased ?

(6,3½)

6. (a) For a given set of observation (x_1, y_1), (x_2, y_2)---(x_n, y_n), fit the models (i) $Y = \beta_1 X + \in$ and (ii) $Y = \beta_0 + \beta_1 X + \in$. Which of the above models is appropriate for the given set of data ? How will you justify your choice ? Give atleast three reasons.

(b) Consider a simple linear progression model $Y = \beta_0 + \beta_1 X + \epsilon$ when X and Y are jointly distributed random variables, but the form of the distribution is unknown. What are the conditions that need to be satisfied so that regression results hold good. (6½,3)