This question paper contains 4 printed pages.]

1432 *Your Roll No. ....*

B.Sc. (Hons.) / III **A**

STATISTICS – Paper XXIV

(Stochastic Processes)

(Admissions of 1999 and onwards)

Time : 2 Hours *Maximum Marks : 38*

(Write your Roll No. on the top immediately on receipt of this question paper.)

*Attempt **four** questions in all, selecting **two** questions from each section.*

Section I

1. (a) Find the covariance function of $\{Y_n\}$ given by $Y_n = a_0X_n + a_1X_{n-1} + \ldots + a_kX_{n-k}$, $n = 1,2 \ldots.$ where a_i' are constants and X_n's are uncorrelated random variables.

(b) Let

$$Y_t = \sum_{n=1}^{N} C_n e^{i\lambda_n t}, i = \sqrt{-1}, \text{ where } \lambda_1, \lambda_2, \ldots, \lambda_N,$$

are real constants, and $C_n's$ are uncorrelated random variables with zero mean and $\text{Var}(C_n) = \sigma_n^2$. Find if the process is stationary ?

[P.T.O.

(c) In a sequence of Bernoulli trials, let u_n be the probability that the combination SFS occurs for the first time at the nth trial. i.e. a success at $(n\text{-}2)^{th}$ trial. followed by a failure at $(n\text{-}1)^{th}$ trial and a success at n^{th} trial. Find the generating function, the mean and the variance of $\{u_n\}$.

3, 3½, 3

2. (a) Define an irreducible Markov chain. Show that, in an irreducible Markov chain, all the states are of same type.

(b) Consider the Markov chain with t.p.m.

$$\begin{array}{c} \\ 1 \\ 2 \\ 3 \\ 4 \end{array}\begin{array}{c} \begin{array}{cccc} 1 & 2 & 3 & 4 \end{array} \\ \begin{bmatrix} \frac{1}{3} & \frac{2}{3} & 0 & 0 \\ 1 & 0 & 0 & 0 \\ \frac{1}{2} & 0 & \frac{1}{2} & 0 \\ 0 & 0 & \frac{1}{2} & \frac{1}{2} \end{bmatrix} \end{array}$$

Examine whether the chain is ergodic.

5, 4½

3. (a) Prove that if the intervals between successive occurrences of an event E are independently distributed exponentially with common mean $\frac{1}{\lambda}$, then the event E has Poisson process as its counting process.

(b) Prove that the interval between two successive occurrences of a Poisson process $\{N(t), t \geq 0\}$ with parameter λ has a negative exponential distribution with mean $\frac{1}{\lambda}$. 5, 4½

Section II

4. (a) In a G.W. branching process with population size X_n at n^{th} generation, show that

$$E(X_{n+r} | X_n) = X_n \mu^r . r.n = 0,1,2....$$

where $\mu = E(X_1)$

(b) Define the probability of ultimate extinction and state the conditions for ultimate extinction.

Let

$p_0 = \alpha$; $p_1 = 1 - \alpha - \beta$; $p_2 = \beta$ and $p_k = 0 \; \forall k \neq 0, 1, 2$

Find the conditions under which the population will ultimately be extinct.

5, 4½

5. (a) In a single server model with finite system capacity N, show that the generating function P(s) of the number of customers in the system is given by (with usual notations)

$P(s) = p_0 (1 - \rho s)^{-1} - \rho p_N s^{N+1} (1 - \rho s)^{-1}$

Hence obtain an expression for p_n.

(b) For $(M/M/1)$ system with infinite system capacity, obtain Var(n) and Var(m), where m is the random variable denoting the queue length and n is the number of customers in the system. Show that

$$\text{Var}(m) + \text{Var}(s) = \text{Var}(n)$$

where s is the random variable denoting the service time.

5, 4½

6. (a) State the classical ruin problem of a gambler. Find an expression for the generating function of the duration of the game which is expected to be finite.

(b) Find the optimal replacement policy in case of items that deteriorate with time in case when the money has the time value. State the assumptions you make.

5½, 4