This question paper contains 4 printed pages.]

1428 *Your Roll No.* ....

B.Sc. (Hons.) / III **A**

STATISTICS – Paper XX

(Statistical Inference II)

(Admissions of 1999 and onwards)

Time : 2 Hours *Maximum Marks : 38*

(Write your Roll No. on the top immediately on receipt of this question paper.)

*Attempt **four** questions in all, selecting **two** questions from each Section.*

Section I

1. (a) Explain the following terms :

(i) errors of first and second kind;

(ii) simple and composite hypotheses;

(iii) the best critical region.

(b) Let p be the probability that a coin will fall head in a single toss. In order to test $H_0 : p = \frac{1}{2}$ against

[P.T.O.

$H_1 : p = \frac{3}{4}$, the coin is tossed 5 times and H_0 is rejected, if more than 3 heads are obtained. Find the probability of type I error and power of the test.

5½,4

2. (a) State and prove Neyman-Pearson lemma.

(b) Let $X_1, X_2, \ldots X_n$ be a random sample from $N(\mu,\theta)$, μ being known and θ being unknown. Obtain the best critical region of size α for testing $H_0 : \theta = \theta_0$ against $H_1 : \theta = \theta_1$ $(\neq\theta_0)$. Hence obtain the power function of the test.

5, 4½

3. Describe Wald's S.P.R.T., its O.C. and A.S.N. functions, construct S.P.R. test for testing $H_0 : \theta = \theta_0$ against $H_1 : \theta=\theta_1$ $(0<\theta_0\theta_1)$ on the basis of a random sample drawn from Pareto distribution with density function.

$$f(x, \theta) = \frac{\theta a^{\theta}}{x^{\theta+1}}, x \geq a.$$

Also obtain its O.C. and A.S.N. functions.

9½

Section II

4. Discuss the method of construction of likelihood ratio test and state its important properties. Consider n Bernoullian trials with probability of success p for each trial. Derive the likelihood ratio test for testing $H_0 : p = p_0$ against $H_1 : p > p_0$.

9½

5. (a) Describe Mann-Whitney-Wilcoxon test. How is the test carried out when

(i) there are ties;

(ii) sample sizes are large ?

(b) Let $x_1, x_2, ..., x_n$ be the observed values of n random variables $X_1, X_2, ..., X_n$. Develop a test for the hypothesis that the random variables are *i.i.d.*

5½,4

6. (a) Name a nonparametric test which is alternative to the one-way analysis of variance for testing that K independent samples are drawn from different populations. Describe its method and function in detail.

(b) Develop the Bayesian method for testing the point null hypothesis $H_0 : \theta = \theta_0$ against $H_1 : \theta \neq \theta_0$ and show that Bayes factor is given by :

$$B = \frac{p(\underset{\sim}{x}/\theta_0)}{p_1(\underset{\sim}{x})},$$

Where $p(\underset{\sim}{x})$ = predictive density of the vector $\underset{\sim}{x} = (x_1, x_2, \ldots x_3)$ and p_1, $(\underset{\sim}{x})$ = predictive density under the alternative hypothesis. Further show that, if $t = t\ (\underset{\sim}{x})$ is sufficient for $\underset{\sim}{x}$ given θ, Bayes factor is

$$B = \frac{p(t \mid \theta_0)}{p_1(t)}.$$ 5½, 4