[This question paper contains 4 printed pages.]

1056 *Your Roll No.* ....

B.Sc. (Hons.) / III **C**

STATISTICS – Paper XIX

C–221 : (Statistical Inference – I)

(Admissions of 1999 and onwards)

Time : 2 Hours *Maximum Marks : 38*

(Write your Roll No. on the top immediately on receipt of this question paper.)

Attempt **four** *questions in all, selecting* **two** *questions from each Section.*

SECTION I

1. (a) What do you understand by point estimation? When would you say that estimate of a parameter is good? In particular, discuss the concepts of consistency and unbiasedness. Give an example to show that a consistent estimator need not be unbiased.

   (b) Show that in random sampling from $N(\mu, \sigma^2)$, where both μ and σ^2 are unknown, the estimator of the form aS^2 for σ^2 which has the smallest mean square error is

$$\frac{n-1}{n+1}S^2 \qquad (5\tfrac{1}{2},4)$$

2. (a) If T_1 is an MVU estimator for θ and T_2 is any other unbiased estimator for θ with efficiency e, then prove that correlation-coefficient between T_1 and T_2 is $\sqrt{e}$.

(b) Given $f(x,\theta) = \begin{cases} \frac{1}{\theta}; & 0 < x < \theta, \theta > 0 \\ 0, & \text{elsewhere,} \end{cases}$

compute the reciprocal of $nE\left\{\left[\frac{\partial \log f(x,\theta)}{\partial \theta}\right]^2\right\}$ and compare this with the variance of $(n+1)\frac{Y_n}{n}$, where Y_n is the largest observation of a random sample of size n from this distribution. Comment on the result. (5,4½)

3. (a) Define the terms :

(i) Risk function and admissibility of a decision rule.

(ii) Natural conjugate prior

Using Jeffery's rule obtain prior p.d.f. of θ, if X follows $N(\theta,\phi)$, where ϕ is known.

(b) Describe the procedure of obtaining estimators by the method of minimum Chi-square. (5½,4)

SECTION II

4. (a) If T is a complete sufficient statistic for $\gamma(\theta)$ and $E(T) = \gamma(\theta)$ then show that $\phi(T)$ is the unique MVU estimator of $\gamma(\theta)$.

Use this property to obtain the MVU estimator of θ based on a random sample $X_1, X_2, \ldots, X_n$ from a bernoulli distribution with parameter θ.

(b) Let the random variables X and Y have the joint p.d.f.

$$f(x,y) = \begin{cases} \left(\dfrac{2}{\theta^2}\right)e^{-\frac{x+y}{\theta}} ; & 0 \le x \le y < \infty, \\ 0 , & \text{elsewhere}. \end{cases}$$

Obtain the expected value of $X + \theta$ and compare the variance of $X + \theta$ with that of Y. (5,4½)

5. (a) Prove that under regularity conditions to be stated by you, any consistent solution of the likelihood equation provides a maximum of the likelihood with probability approaching unity as the sample size tends to infinity.

(b) Obtain the maximum likelihood estimator of θ in sampling from a distribution having p.d.f.

$$f(x,\theta) = \begin{cases} \theta x^{\theta-1}; & 0 \le x \le 1, \theta > 0 \\ 0, & \text{elsewhere} \end{cases}$$

Also obtain the MVU estimator for θ. (5,4½)

6. (a) Describe the general method of constructing the confidence interval for large samples. If $X_1, X_2, \ldots, X_n$ is a random sample of size n from an exponential distribution with parameter θ, obtain a 95% confidence interval for θ when n is large.

(b) Describe the method of moments. For the double Poisson distribution

$$P(x) = P[X = x] = \frac{1}{2}\frac{e^{-m}m_1^x}{x!} + \frac{e^{-m}m_2^x}{x!}, \quad x = 0,1,2,\ldots$$

Show that the estimates for m_1 and m_2 by the methods of moments are

$$\mu_1' \pm \sqrt{\mu_2' - \mu_1' - (\mu_1')^2}\ .$$ (4½,5)