[This question paper contains 4 printed pages.]

1057 *Your Roll No.* ....

B.Sc. (Hons.) / III **C**

STATISTICS – Paper XX

C–222 : (Statistical Inference – II)

(Admissions of 1999 and onwards)

Time : 2 Hours *Maximum Marks : 38*

(Write your Roll No. on the top immediately on receipt of this question paper.)

Attempt **four** *questions in all, selecting* **two** *questions from each Section.*

SECTION I

1. (a) Explain the following terms

(i) Statistical hypothesis. Simple and composite hypothesis.

(ii) One tailed and two tailed tests.

(iii) Bayes factor and its applications in decision analysis.

P.T.O.

(b) Let X_1, X_2 be a random sample from a distribution with p.d.f.

$$f(x;\theta) = \begin{cases} \frac{1}{\theta} e^{-x/\theta}, & 0 < x < \infty, \theta > 0, \\ 0, & \text{elsewhere}. \end{cases}$$

The critical region for testing $H_0 : \theta = 2$ against $H_1 : \theta = 1$ is given by

$$W = \{(x_1, x_2)/x_1 + x_2 \geq 9.5\}$$

Find (i) Significance level. (ii) Power of the test.

(5,4½)

2\. (a) Let $X_1, X_2, \ldots, X_n$ be a random sample from a distribution which has p.d.f.

$$f(x;\theta) = \begin{cases} \theta x^{\theta-1}, & 0 < x < 1, \\ 0, & \text{elsewhere}. \end{cases}$$

Derive UMP test of level α for testing

(i) $H_0 : \theta = \theta_0$ against $H_1 : \theta > \theta_0$ and

(ii) $H_0 : \theta = \theta_0$ against $H_1 : \theta < \theta_0$.

Show that these tests may be defined in terms of χ^2 statistic with 2n d.f.

(b) Define UMPU critical region. Prove that if W is an MP region for testing $H_0 : \theta = \theta_0$ against $H_1 : \theta = \theta_1$ then it is necessarily unbiased.

(5½,4)

3. Describe Likelihood Ratio test. Develop L R test for testing $H_0 : \theta = \theta_0$ against various alternatives in $N(\theta, \sigma^2)$ where σ^2 is unknown. Mention properties of L R test. (9½)

SECTION – II

4. (a) Let X be a random variable having p.d.f. $N(\mu, \theta)$, where μ is known. Develop S.P.R.T. for testing $H_0 : \theta = \theta_0$ against $H_1 : \theta = \theta_1$ $(> \theta_0)$. Also obtain the expressions for its O.C. and A.S.N. functions.

(b) For the S.P.R.T. of strength (α^1, β^1) and for given

$$A = \frac{1-\beta}{\alpha} \text{ and } B = \frac{\beta}{1-\alpha} \text{; prove that}$$

(i) $\alpha^1 \leq \frac{\alpha}{1-\beta}$, $\beta^1 \leq \frac{\beta}{1-\alpha}$

(ii) $\alpha^1 + \beta^1 \leq \alpha + \beta$ (5½,4)

5. (a) Develop Mann-Whitney-Wilcoxon test for testing whether two given samples are drawn from the same continuous populations. Carry out the above test for small as well as large samples.

(b) Develop Kruskal-Wallis test for deciding whether K independent samples are from different populations. (5,4½)

6. (a) Develop median test for testing the hypothesis that two independent groups differ in central tendencies.

(b) Explain Bayesian method for testing :

(i) Simple $H_0 : \theta = \theta_0$ against simple $H_1 : \theta = \theta_1$.

(ii) Composite $H_0 : \theta \in \Theta_0$ against composite $H_1 : \theta \in \Theta_1$ (4½,5)